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TECHNICAL REPORT ARBRL-TR-02219

SELECTION AND VALIDATION OF A MULTIPLE FIBER MODEL

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March 1980

US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
BALLISTIC RESEARCH LABORATORY
ABERDEEN PROVING GROUND, MARYLAND

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The objective of this test was to verify the exponential exposure model for the vulnerability to carbon fiber of equipment with contact distances less than one fiber length and to develop a mathematical model for contact distances greater than one fiber length. The need for this work arises from the fiber length spectra observed during open air tests at China Lake and Dugway Proving Grounds. The exposure model developed and validated for all failures was the following Weibull distribution

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$$F_n(E) = 1 - \exp[-\hat{\alpha} E^n]$$

where n is the number of fibers needed to bridge the electrode gap and $1/\hat{\alpha}$ is the maximum likelihood estimate of the mean exposure to failure. The exponential distribution is a special case of the Weibull where n = 1.

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I. INTRODUCTION

The objective of this task was to verify the exponential exposure model for the vulnerability of equipment to carbon fibers with contact distances less than the fiber length and to develop a mathematical model for contact distances greater than one fiber length. The need for this work arises from the fiber length spectra which have been documented by the TRW results from the open air tests at China Lake.[1] The fiber length spectra seen in these tests have their peak in the region of 3mm. If the lengths of accidentally released fibers are short (<3mm), it would be beneficial to have a model which would predict vulnerability for equipment with vulnerable points greater than one fiber length.

II. SELECTION OF A MODEL

In developing a mathematical model to describe the kill probabilities due to carbon fibers, it was found that if a single fiber is capable of effecting a kill, that is the gap between electrical contacts is less than or equal to one fiber length, then the probability of kill due to a single fiber is much higher than the probability of kill due to any multiple fiber event. Most systems tested have been susceptible to single fiber kill where the fibers were 3.5 to 15mm in length.

The question to be addressed here is; if a single fiber cannot effect a kill because the gap width exceeds the fiber length, what model should be used to describe the probability of kill due to a multiple fiber event?

Let F(x) be the distribution function of the time-to-failure random variable X, and let f(x) be its probability density function. Then the hazard rate, h(x), is defined as

$$h(x) = \frac{f(x)}{1-F(x)}. \qquad 2.1$$

Here 1-F(x) is called the reliability at time x. The hazard rate, which is a function of time, has a probabilistic interpretation; namely, h(x) dx represents the probability that a device of age x will fail in the interval (x, x+dx), or

$$h(x) = \lim_{\Delta x \to 0} \left[\frac{P\left\{\text{a device of age x will fail in the interval (x, x+dx)/it has survived up to x}\right\}}{\Delta x} \right].$$

[1] *"Data Reduction and Analysis of Graphite Release Experiments", Paul Lieberman, et. al., TRW Defense and Space System Group, 1979.*

On the basis of physical considerations, h(x) will be chosen for a particular device. Given the choice of h(x) and assuming $F(0^-) = 0$ and $F(\infty) = 1$, f(x) and F(x) can be determined as follows:

By definition,
$$\int_0^x f(s)ds = F(x)$$

and
$$\frac{dF(x)}{dx} = f(x).$$

Now from equation 2.1
$$h(x)dx = \frac{dF(x)}{1-F(x)}$$

or
$$\int_0^t h(x)dx = -\ln\left[1-F(x)\right]\Big|_0^t .$$

Thus,
$$\ln\frac{1-F(t)}{1-F(0)} = -\int_0^t h(x)dx$$

or
$$1-F(t) = \exp\left[-\int_0^t h(x)dx\right]$$

and
$$F(t) = 1-\exp\left[-\int_0^t h(x)dx\right] \qquad 2.2$$

Taking the derivatives
$$f(t) = h(t)\exp\left[-\int_0^t h(x)dx\right] .$$

The above derivation of the distribution and density functions using the hazard rate concept is presented by Mann, Schafer, and Singpurwalla in Methods for Statistical Analysis of Reliability and Life Data.

The devices for which we wish to determine hazard rates can be divided into categories based on the number of fibers needed to effect a kill.

For the case in which a single fiber can effect a kill, the hazard rate, $h_1(x)dx$, is the probability that a fiber arrives at a vulnerable site in the time interval (x, x+dx). It seems reasonable to assume that the probability of a fiber arriving in the interval (x, x+dx) would be proportional to the concentration (C) at the site, indicated by the position vector $\overline{r}$, in the time dx.

Let

$$h_1(x)dx = \alpha\, C(\bar{r},x)dx$$

Substituting in equation 2.2, we have

$$F_1(t) = 1\text{-}\exp\left[-\alpha\int_0^t C(\bar{r},x)dx\right] .$$

By definition, the time integral of concentration is exposure (E), i.e.

$$\int_0^t C(\bar{r},x)\ dx = E(\bar{r},x).$$

We now have the distribution as a function of exposure, which is time dependent,

$$F_1(E) = 1\text{-}\exp\left[-\alpha E(\bar{r},x)\right] .$$

Using the method of maximum likelihood, the best estimate of α can be computed (Appendix A).

For the case in which two fibers are needed to effect a kill, the hazard rate, $h_2(x)dx$, is the probability that the second fiber arrives in the interval $(x,x+dx)$ given one fiber is at the site at time x. As for a single fiber kill, we again assume that the probability of a fiber arriving in the interval $(x,x+dx)$ is proportional to the concentration in the time interval dx. We are assuming that the probability of a fiber being located at the site at time x is proportional to the time integral of concentration, exposure, up to time x, therefore

$$h_2(x)dx = \beta EC\ dx$$

Substituting in equation 2.2, we have

$$F_2(t) = 1\text{-}\exp\left[-\beta\int_0^t CE\ dx\right]$$

$$= 1\text{-}\exp\left[-\ \beta E^2\right]$$

It is understood that concentration and exposure are both time and position dependent. We can again write this distribution as a function of exposure, as

$$F_2(E) = 1\text{-}\exp\left[-\beta E^2\right]$$

and the best estimate of β found using the method of maximum likelihood (Appendix A).

This process can be generalized to the case requiring n fibers to effect a kill. The hazard rate, $h_n(x)dx$, would be the probability the n^{th} fiber arrives in the interval (x, x+dx) given n-1 fibers located at the site at time x. If the probability of each of the n-1 fibers located at the site at time x is proportional to the exposure up to time x, and each fiber arrives independently, then the probability of n-1 fibers being located at the site at time x is proportional to $E(\bar{r},t)^{n-1}$. The probability of the n^{th} fiber arriving in (x, x+dx) is again proportional to the concentration for the period dx. Therefore

$$h_n(x)dx = \gamma\ E^{n-1}\ C\ dx$$

Substituting in equation 2.2 and writing as a function of exposure, we have

$$F_n(E) = 1\text{-exp}\ [\ -\gamma \int_0^t E^{n-1}\ Cdx]$$

$$= 1\text{-exp}\ [-\gamma E^n]$$

The maximum likelihood estimate can again be found for γ (Appendix A).

This distribution function is the Weibull distribution with shape parameter n and scale parameter γ. For a single fiber kill, n=1, this gives a special case of the Weibull distribution, the exponential distribution.

III. EXPERIMENTAL FACILITIES

A. Exposure Chamber

The exposure chamber used to measure multifiber vulnerability was a modified S-280 van. This chamber is composed of two separate sections, the actual test chamber and an anteroom. The fiber exposure section is a room 3.5m x 1.9m x 2m with a window in the side for easy observation of the experimental area during the exposure. The test chamber has twelve muffin fans which are Variac controlled, and used to maintain a homogenous distribution of fibers throughout the chamber. The use of these fans has produced a homogenous concentration which varied between 10^3 and 10^4 fibers/m^3 for 20 minutes without the addition of any fibers.

The low airflow allows the assumption that the carbon fiber velocity is close to settling velocity. Therefore, the experimental results are not biased by a high fiber velocity.

The anteroom (1.9m high x 2m wide x 2.2m long) is used for target and fiber preparation. This room is equipped with a sticky paper pad and a large vacuum cleaner to prevent the migration of carbon fibers out of the experimental area. For futher information concerning the exposure chamber, see reference 2.

B. Fiber Dispenser

Fiber dispensing is accomplished by an instrument designed and fabricated at the BRL[3]. This dispensing system is housed in a vertical cylindrical container 20cm in diameter and 1m high and is located in the exposure chamber. The exit port is tapered in such a way as to increase fiber velocity at that point and eject single fibers into the room. There is a constant airflow through the dispenser which is a factor of two greater than the fiber fall velocity (2.5cm/sec). At the base of the fiber container is an outlet which emits a short burst of high pressure air at regular increments. Both the air burst duration and repetition are electronically controlled. The short burst of air serves to lift a large clump of fibers. The single fibers, because of the steady upward flow of air, will continue up the container and be ejected. The clumps, because of their greater fall velocity, will settle back to the bottom and be reelevated by one of the following bursts. The fibers used in the dispenser are precut. The fibers dispensed are 90 per cent single fibers with no noticeable length breakup. Approximately 1gm of precut fiber will produce an exposure of 1 to 2 x 10^7 fiber-sec/cubic meter ($f \cdot s/m^3$) of 7.5mm long fibers in the exposure chamber during a 30 minute trial.

C. Fiber Detection

There were two methods of fiber detection used during this task. The active method, which was the BRL ball gauge, was used to measure the exposure at the time of failure. The ball gauge uses electronic circuitry to count the number of fibers present which are greater than one half the nominal length being dispensed. The data was stored in a multichannel analyzer in the multichannel scaling mode. Thus, the record was the concentration as a function of time, the sum of which is exposure.

The passive method of detection was a sticky paper sample 39mm square which was used to measure the fiber length sprectrum dispensed. Because the purpose of the work was to express the probability of failure as a function of exposure with varying fiber lengths,

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[2] *A. Croce and G. A. Dunn, "Filter Evaluations - Project HAVE NAME", ECOM Research and Development Technical Report No. ECOM-4286, January 1975, SECRET.*

[3] *Private Communication, Neil Wolfe of BRL.*

the concentration and the fiber length had to be carefully monitored. A typical concentration profile can be seen in Figure 1 and a fiber length spectrum in Figure 2. For further information about the detectors. see reference 4.

IV. TEST DESCRIPTION

A. Test Objective

The objective of a test was to measure simultaneously the exposure at the time of the failure for the different electrode gaps. These tests were performed using 7.5mm Hercules HMS fibers. The fiber velocity was settling velocity (2.5cm/sec), and the targets were oriented at 0, 45, and 90 degrees to the horizontal.

B. Target Description

1. Five Space Board

The contact spacing of interest were L/3, L/2, L, 2L, and 4L, where L is the nominal fiber length being dispensed (7.5mm). The five different space targets were located on a phenolic board, 16cm wide and 25cm long. Figure 3 shows the board. Each target consists of two strands of 20 gauge wire, 5cm long separated by the required distance. To insure no interaction between targets, a wooden insulator, 1.25cm high, was used to separate targets. During each trial there were four boards placed in the exposure chamber.

2. Three Space Board

During the first series of tests, the data showed an exposure limit for the large electrode gaps. To investigate this phenomenon, it was decided to construct a target board with very large electrode gap to fiber length ratios, 5L, 10L, and 20L. Figure 4 is a photograph of this board. There are no insulating barriers between targets on these boards because it was constructed in such a way that the adjoining wire elements were at the same electrical potential, and thus, there could be no possibility of any target interaction. These targets were constructed the same, two 5cm lengths of 20 gauge wire. Because these tests were only to check the previous data, the trials were only performed at one angle, 0°.

C. Data Acquisition

As was mentioned earlier, an individual target consisted of two 5cm long strands of 20 gauge wire with the appropriate electrode gap. One electrode was connected to a 100Vdc source and the other to ground through a 9.1Ω resistor. When the electrode gap was bridged, whether by one or many fibers, a strip chart recorder would measure the voltage drop across the resistor. Figure 5 is a block diagram of the measurement circuit.

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[4] *John A. Morrissey, W. I. Brannan, S. C. Thompson, "Calibration of BRL Ball and Sticky Cylinder Detector Systems", Ballistic Research Laboratory Technical Report, ARBRL-TR-02079, June 1978.* (AD #B029204L)

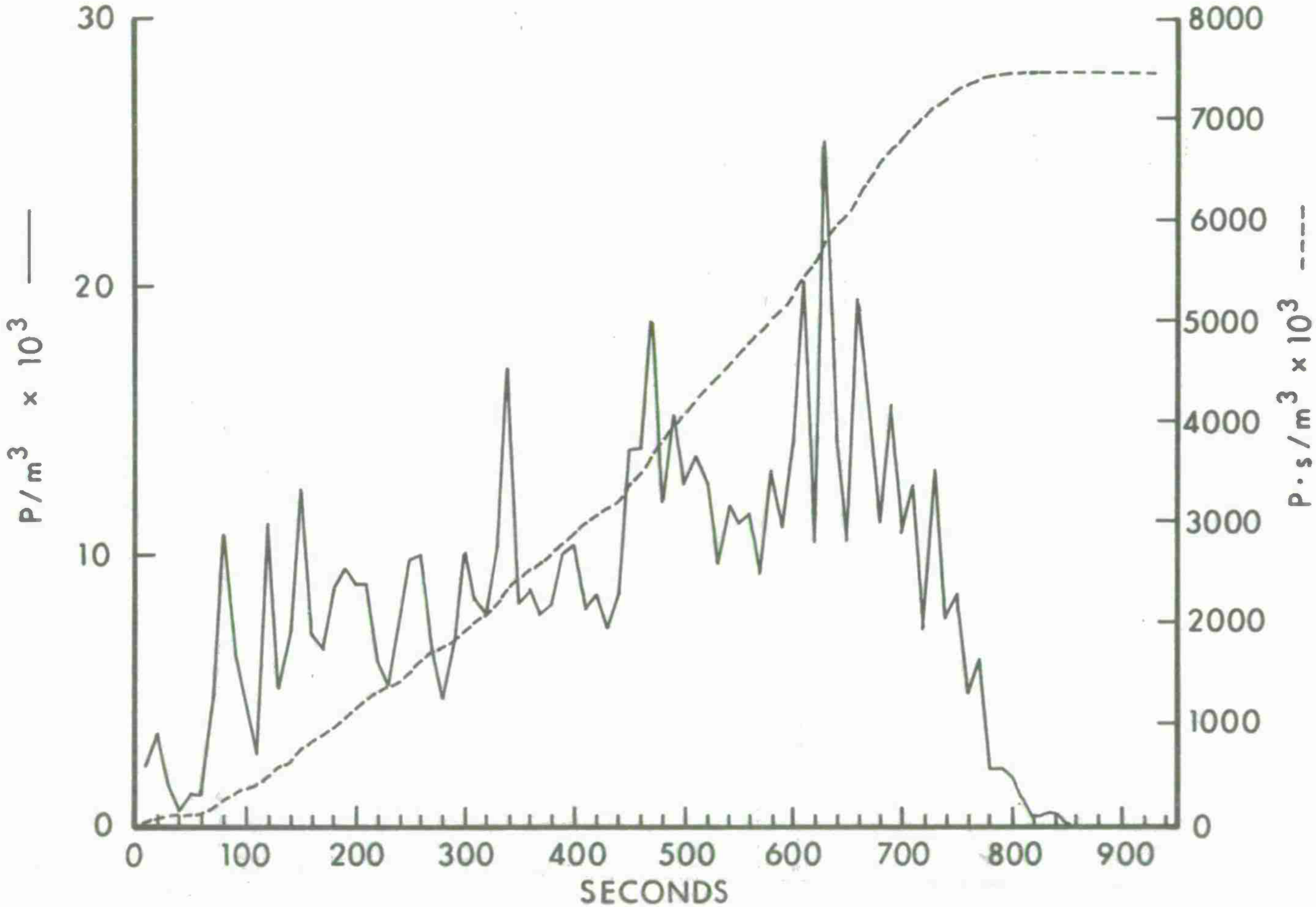

Figure 1. Typical Concentration and Exposure Plot

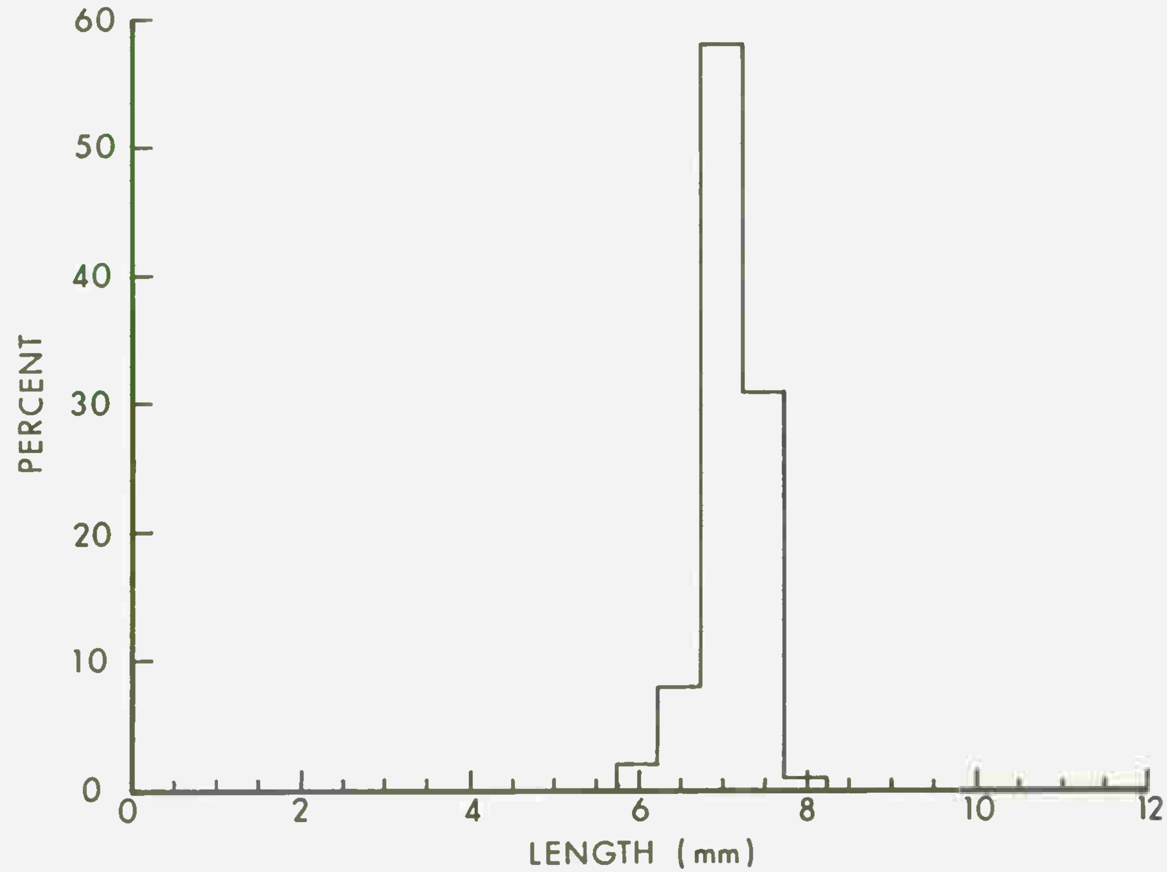

Figure 2. Typical Fiber Length Spectrum

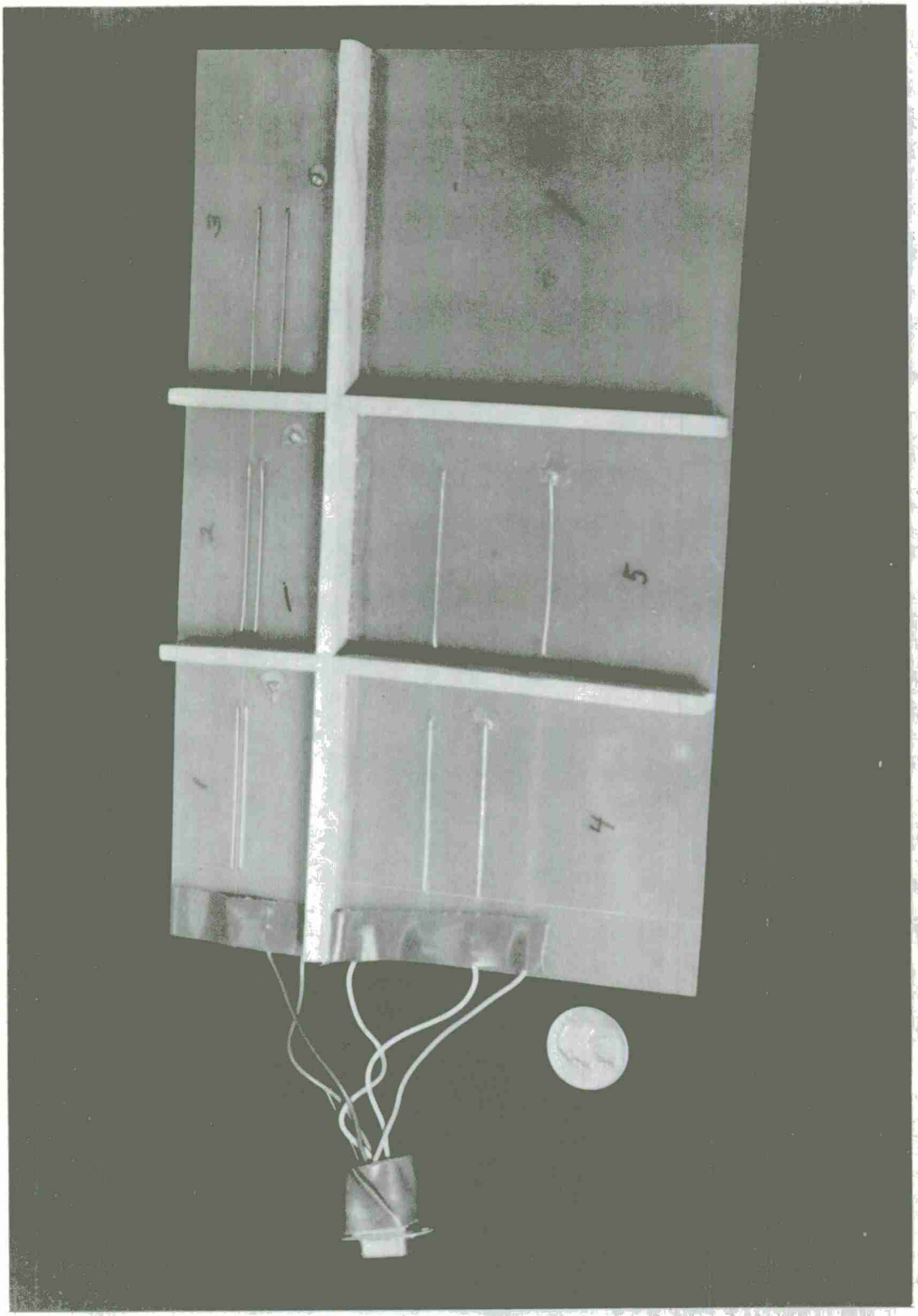

Figure 3. Photograph of Five Target Board

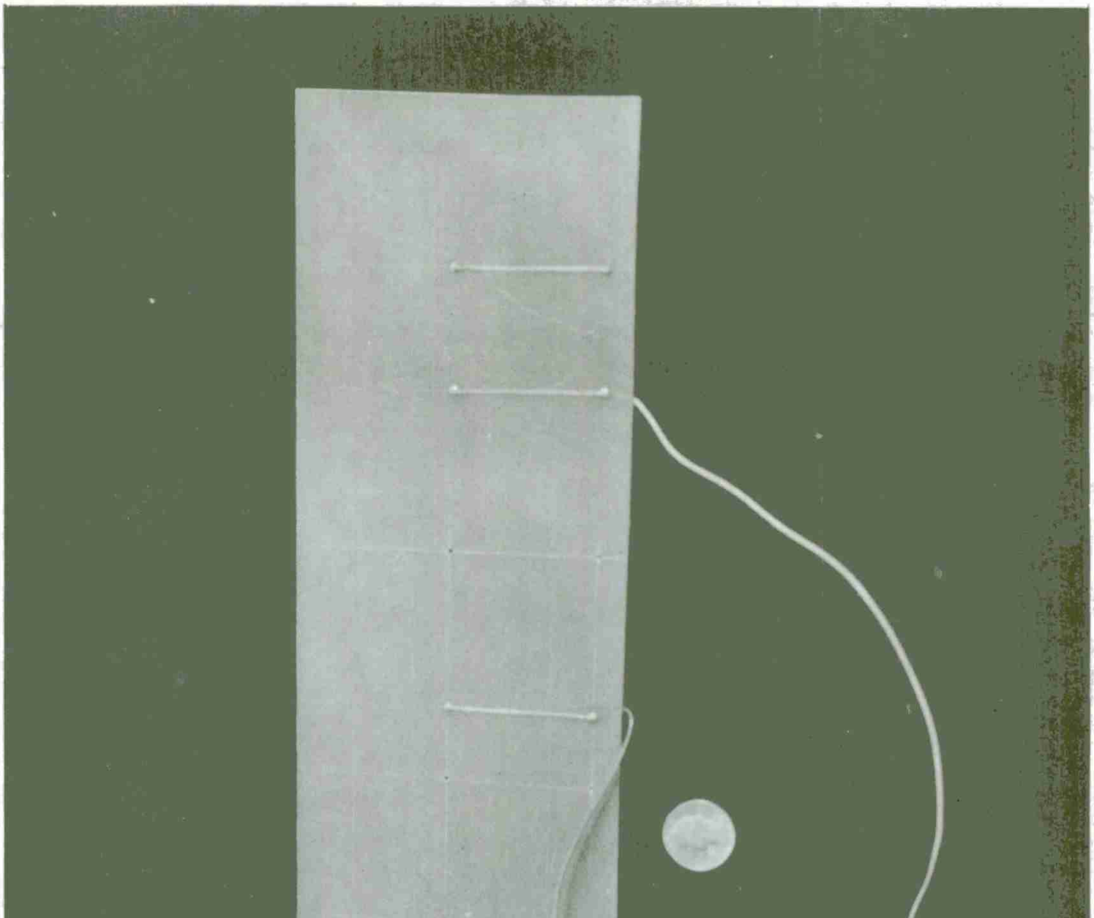

Figure 4. Photograph of Three Target Board

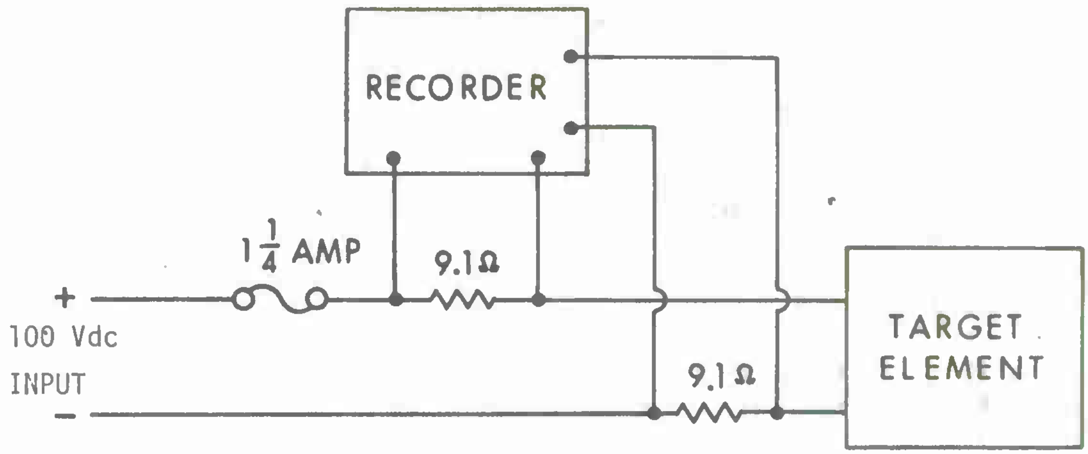

Figure 5. Target Monitoring Circuit

D. Test Procedure

After the target boards had been placed in the chamber, the carbon fiber dispensing began. The fibers used were 7.5mm Hercules HMS, which has a conductivity of 3000 ohms/cm. A data event was defined as a noticeable current spike ($\geq$ 2ma) on both the voltage and ground sides of a target. The exposure was continued until there were two data events at each target, at which time the exposure was terminated.

V. DATA ANALYSIS

The exposures to failure for each test are given in Tables I thru VI. The empirical distribution and theoretical distribution, as derived in Section II, are plotted on Figures 6 thru 23. The single fiber model is used for L/3, L/2, and L. For 2L, 4L, 5L, and 10L the minimum number of fibers needed to bridge the gap is the number of fibers used in the model. A fourteen-fiber model is used for 20L since anything larger overflows the computer. The figures give 1) the orientation of the target - the contact spacing in fiber lengths, 2) the maximum likelihood estimate of the parameter γ, (or α or β) 3) the number of tests and, 4) the Kolmogorov-Smirnov goodness-of-fit statistic T.[5] An asterisk beside the T statistic indicates significance at the α=.05 confidence level.

Thirteen of the eighteen hypothesized distributions fit the data using the Kolmogorov-Smirnov goodness-of-fit test at the 95% confidence level. The five distributions which do not fit are from the data sets[6] L/3, L/2, and 4L at a 45° orientation, 10L and 20L. The model that best fits the data 45°-L/3 and 45°-L/2 is a model using half a fiber (n=.5), shown on Figures 24 and 25. It is possible that the fiber being two or three times longer than the gap enhances the kill probability. However, the four other tests at the same fiber lengths do not indicate an enhanced kill probability.

The data set 45°-4L is best fit using a single fiber model, Figure 26. Since a 7.5mm fiber length and a 30mm gap make this impossible, no explanation is offered. Figure 27 shows the family of curves for the number of fibers, n = 1,2,3,4,5 and 10.

The last three sets of tests having gap widths 5L, 10L, and 20L, were conducted to see if there was an exposure limit for the large electrode gaps. The model that fits each of these data sets best is the five fiber model, Figures 21, 28, and 29. This would indicate that a gap width greater than or equal to five times the fiber length needs an exposure that essentially covers the target. The five fiber model would then be the limiting distribution to describe the situations

[5] *Appendix B gives the distribution of the K-S statistic for the Weibull distribution with scale parameter unknown.*

[6] *The best fit for a set of data is the model having the smallest T value.*

TABLE I. Summary of L/3 Electrode Gap Data

EXPOSURE (f-s/m^3)

	TARGET ORIENTATION		
	0°	45°	90°
Test No.			
1	1.19×10^5	1.03×10^5	1.32×10^6
2	3.69×10^5	1.53×10^5	3.29×10^5
3	3.67×10^5	7.95×10^4	1.48×10^6
4	1.68×10^5	2.0×10^3	1.62×10^6
5	5.87×10^5	2.8×10^6	1.02×10^6
6	5.95×10^4	1.35×10^5	4.79×10^5
7	7.17×10^5	6.5×10^3	8.05×10^5
8	1.14×10^6	6.5×10^3	5.52×10^5
9	4.09×10^5		
10	6.7×10^4		
11	9.9×10^4		
12	4.0×10^3		

TABLE II. Summary of L/3 Electrode Gap Data

EXPOSURE (f-s/m^3)

	TARGET ORIENTATION		
	0°	45°	90°
Test No.			
1	4.35×10^5	5.40×10^6	1.12×10^6
2	2.54×10^5	1.02×10^6	2.19×10^6
3	1.82×10^6	6.5×10^3	1.09×10^7
4	1.68×10^5	9.4×10^4	1.02×10^6
5	5.87×10^5	1.13×10^5	2.77×10^6
6	5.95×10^4	5.6×10^4	1.71×10^6
7	2.15×10^4	3.5×10^2	1.47×10^6
8	4.09×10^5	6.85×10^4	
9	4.52×10^5		
10	1.99×10^5		
11	2.6×10^5		
12	1.94×10^6		

TABLE III. Summary of L Electrode Gap Data

EXPOSURE (f-s/m^3)

	TARGET ORIENTATION		
	0°	45°	90°
Test No.			
1	4.18×10^6	6.75×10^5	1.20×10^7
2	4.28×10^5	1.13×10^5	1.82×10^6
3	9.58×10^5	1.44×10^6	1.43×10^7
4	9.79×10^5	8.27×10^5	6.39×10^6
5	1.52×10^6	6.05×10^6	5.33×10^6
6	7.76×10^5	4.10×10^6	4.70×10^6
7	2.54×10^5	1.63×10^6	8.87×10^6
8	1.45×10^6	9.4×10^4	7.42×10^6
9	8.08×10^5		
10	6.02×10^6		
11	2.98×10^6		
12	1.26×10^6		

TABLE IV. Summary of 2L Electrode Gap Data

EXPOSURE (f-s/m^3)

	TARGET ORIENTATION		
	0°	45°	90°
Test No.			
1	$1.62x10^6$	$9.01x10^6$	$2.1x10^7$
2	$8.19x10^6$	$4.34x10^6$	$4.03x10^7$
3	$1.71x10^6$	$2.85x10^6$	$3.03x10^7$
4	$4.11x10^6$	$6.47x10^6$	$1.55x10^7$
5	$5.21x10^6$	$7.84x10^6$	$2.39x10^7$
6	$3.48x10^6$	$4.89x10^6$	$3.76x10^7$
7	$4.74x10^6$	$3.61x10^6$	$8.9x10^7$
8	$5.25x10^6$	$5.46x10^6$	$7.70x10^7$
9	7.8010^6		
10	$3.38x10^6$		
11	$1.82x10^6$		
12	$2.66x10^6$		

TABLE V. Summary of 4L Electrode Gap Data

EXPOSURE (f-s/m^3)

	TARGET ORIENTATION		
	0°	45°	90°
Test No.			
1	$6.03x10^6$	$1.67x10^7$	$4.03x10^7$
2	$1.74x10^6$	$1.09x10^7$	$5.05x10^7$
3	$4.83x10^6$	$2.85x10^6$	$2.39x10^7$
4	$9.39x10^6$	$6.87x10^6$	$3.76x10^7$
5	$4.38x10^6$	$2.22x10^7$	$8.90x10^7$
6	$5.38x10^6$	$2.15x10^7$	$7.70x10^7$
7	$6.11x10^6$	$6.23x10^6$	
8	$6.33x10^6$	$5.30x10^6$	
9	$6.11x10^6$		
10	$6.40x10^6$		
11	$4.84x10^6$		
12	$3.38x10^6$		

TABLE VI. Summary of Large Electrode Gap Data

EXPOSURE (f-s/m^3)

ELECTRODE GAP SPACINGS

Test No.	5L	10L	20L
1	7.18×10^6	6.25×10^6	6.25×10^6
2	7.59×10^6	6.89×10^6	6.03×10^6
3	9.63×10^6	6.25×10^6	5.89×10^6
4	9.44×10^6	6.63×10^6	6.81×10^6
5	5.46×10^6	5.65×10^6	5.65×10^6
6	8.62×10^6	7.12×10^6	5.77×10^6
7	1.10×10^7	8.97×10^6	8.12×10^6
8	1.02×10^7	8.97×10^6	7.36×10^6
9	9.07×10^6	6.18×10^6	5.28×10^6
10	8.87×10^6	8.77×10^6	7.34×10^6
11	9.36×10^6	5.78×10^6	5.78×10^6
12	8.97×10^6	9.55×10^6	9.24×10^6
13	7.92×10^6	9.36×10^6	6.18×10^6
14	8.77×10^6	9.34×10^6	1.0×10^7

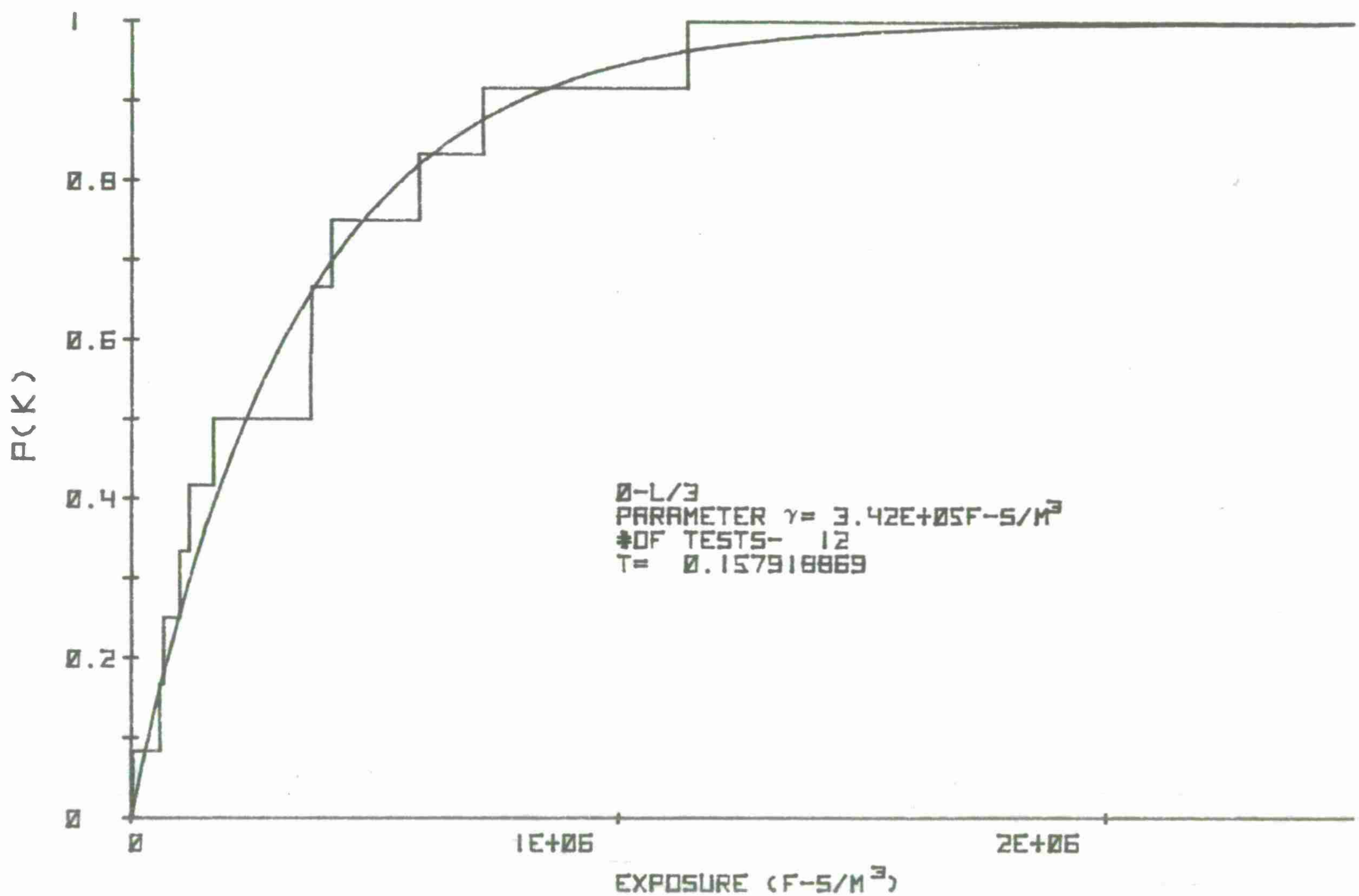

Figure 6. Plot of the 0-L/3 Data and the Theoretical Distribution For a 1 Fiber Model

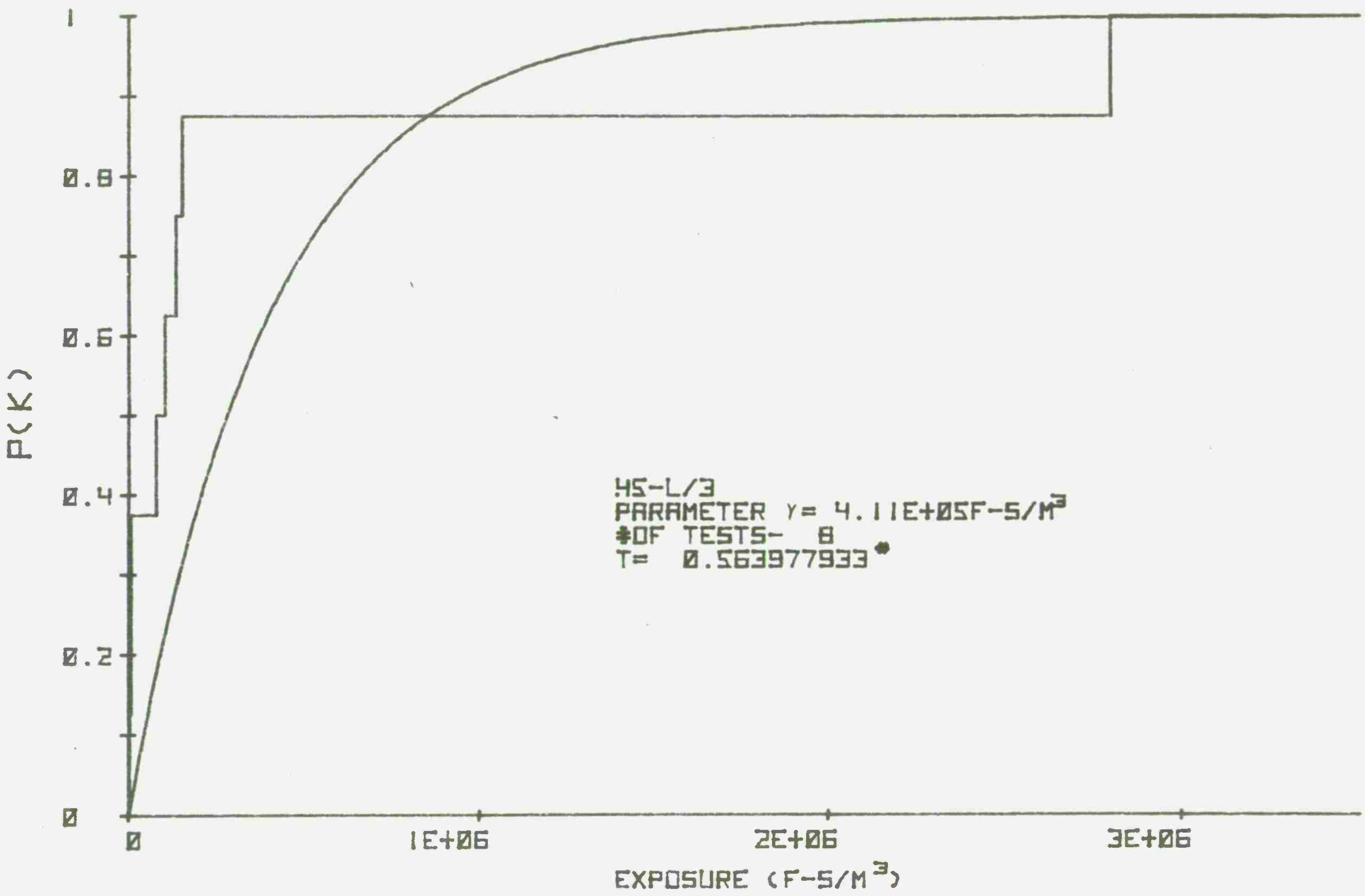

Figure. 7. Plot of the 45-L/3 Data and the Theoretical Distribution For a 1 Fiber Model

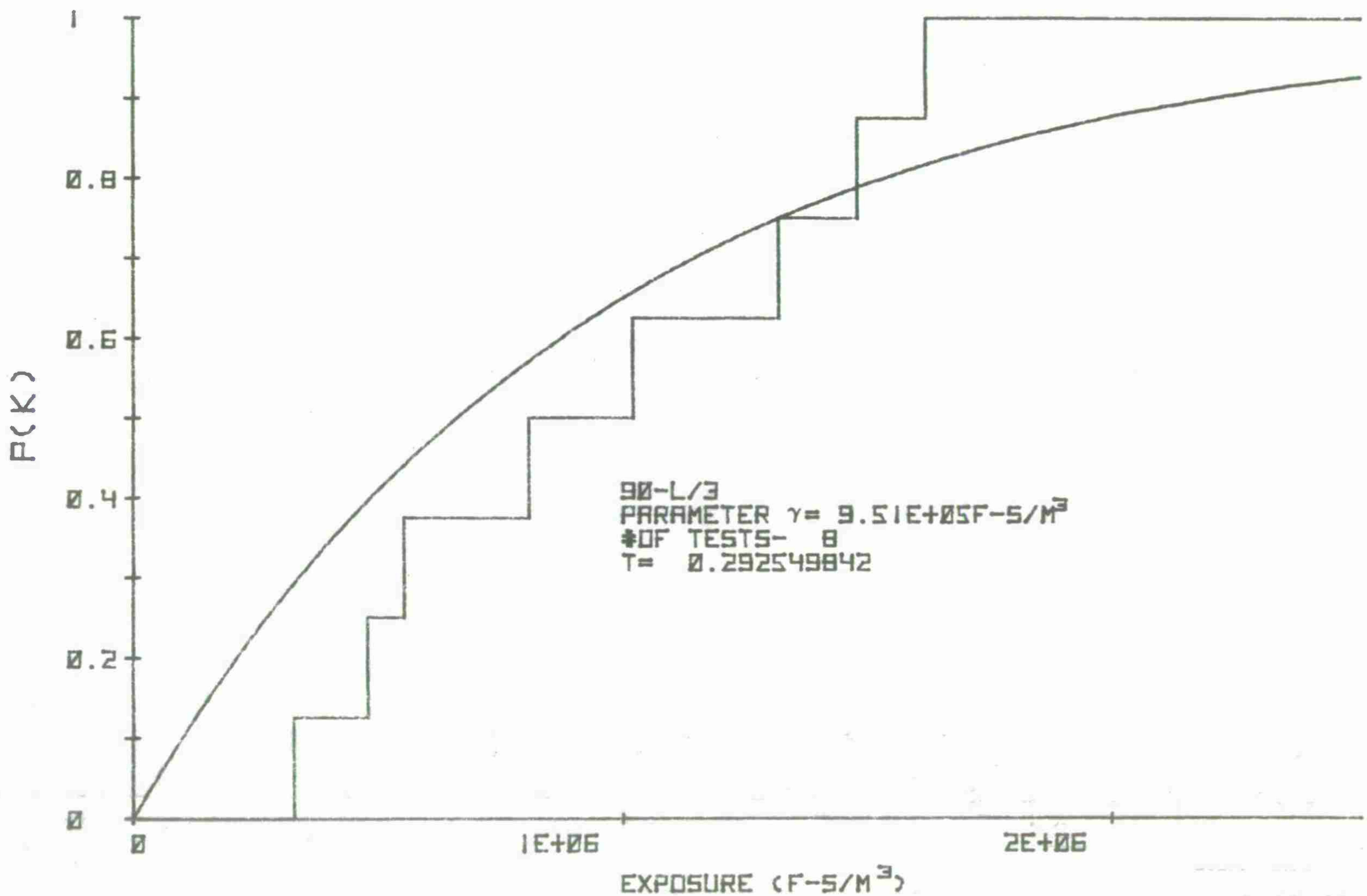

Figure 8. Plot of 90-L/3 Data and the Theoretical Distribution For a 1 Fiber Model

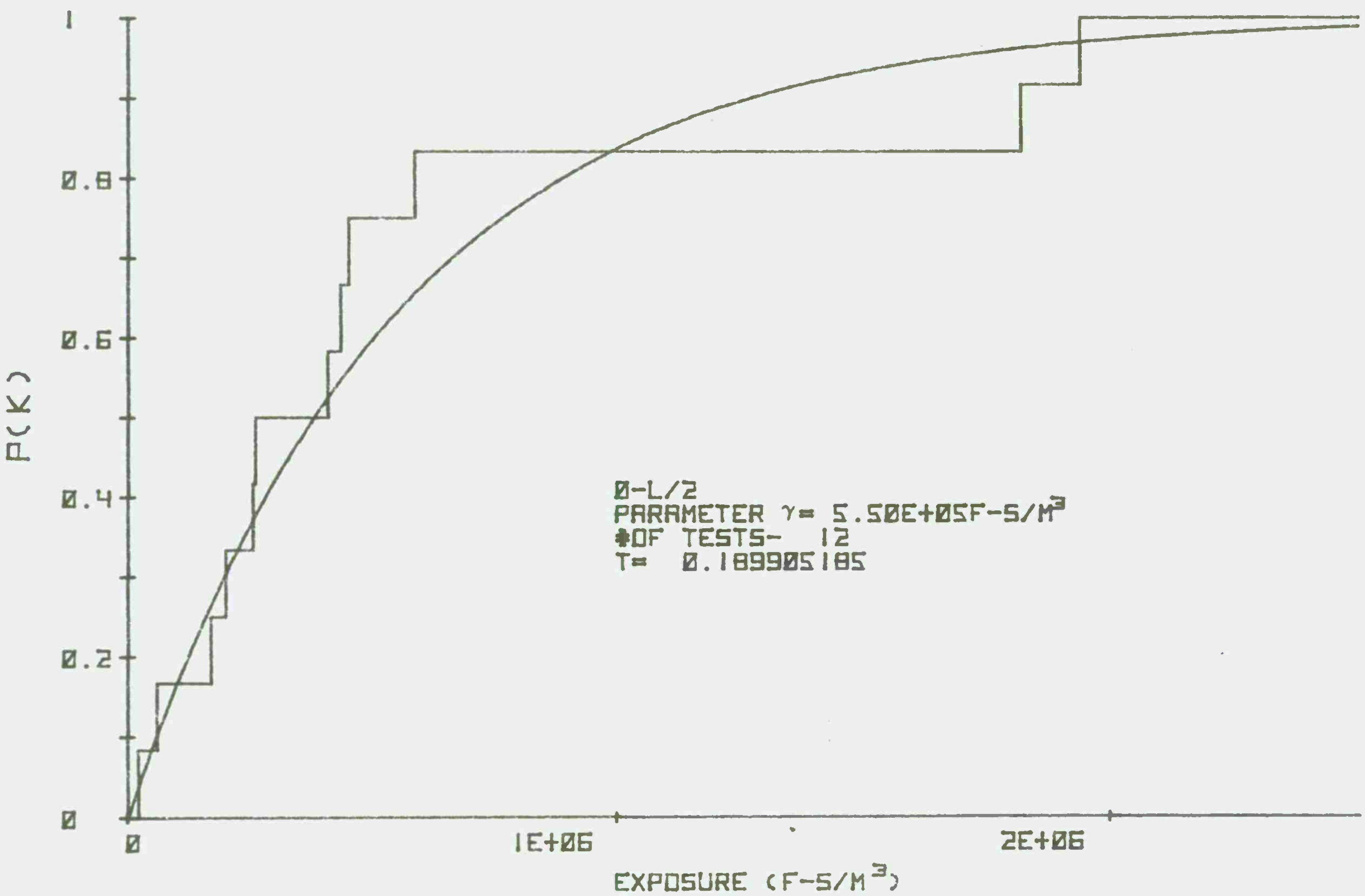

Figure 9. Plot of 0-L/2 Data and the Theoretical Distribution For a 1 Fiber Model

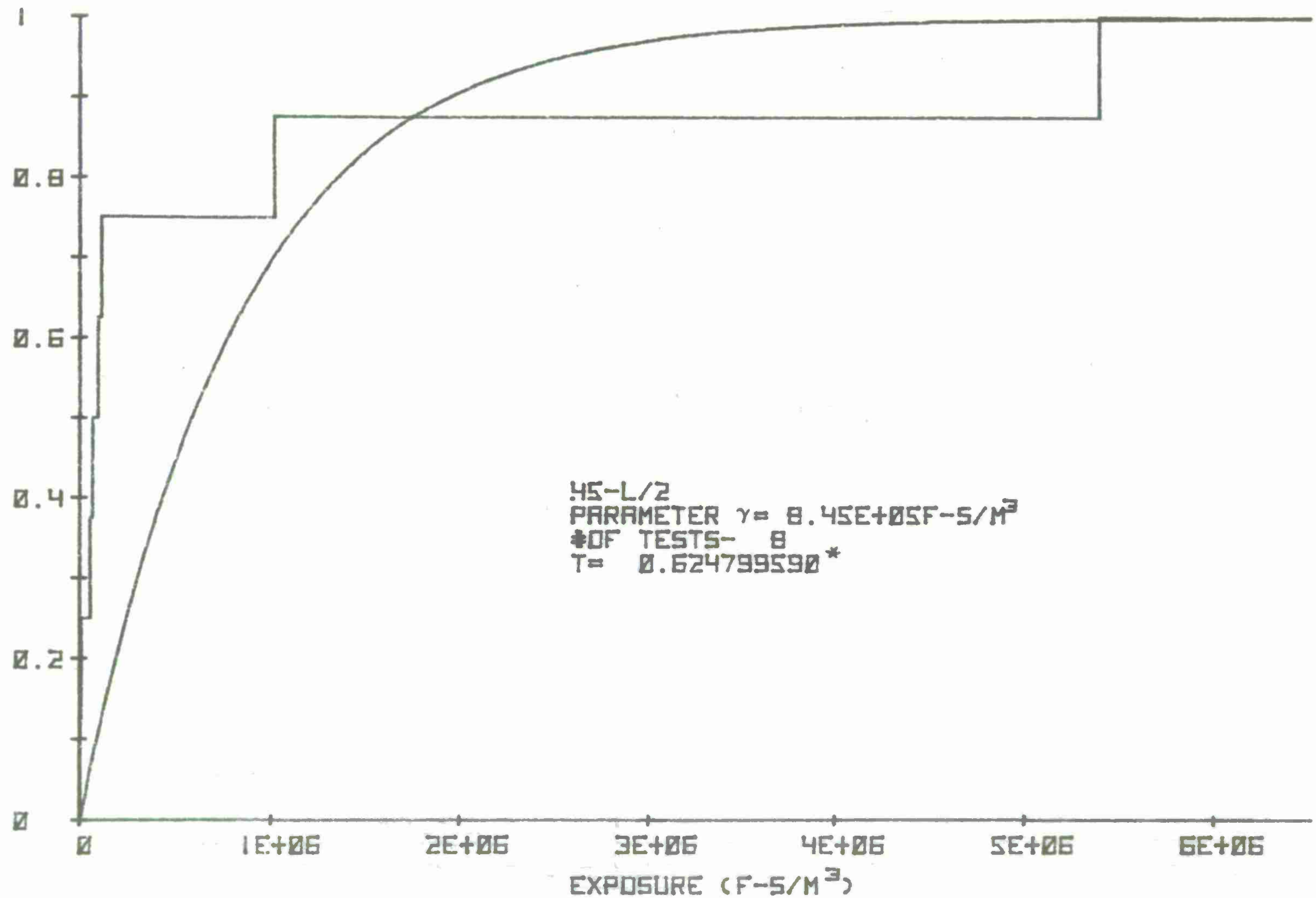

Figure 10. Plot of 45-L/2 Data and the Theoretical Distribution For a 1 Fiber Model

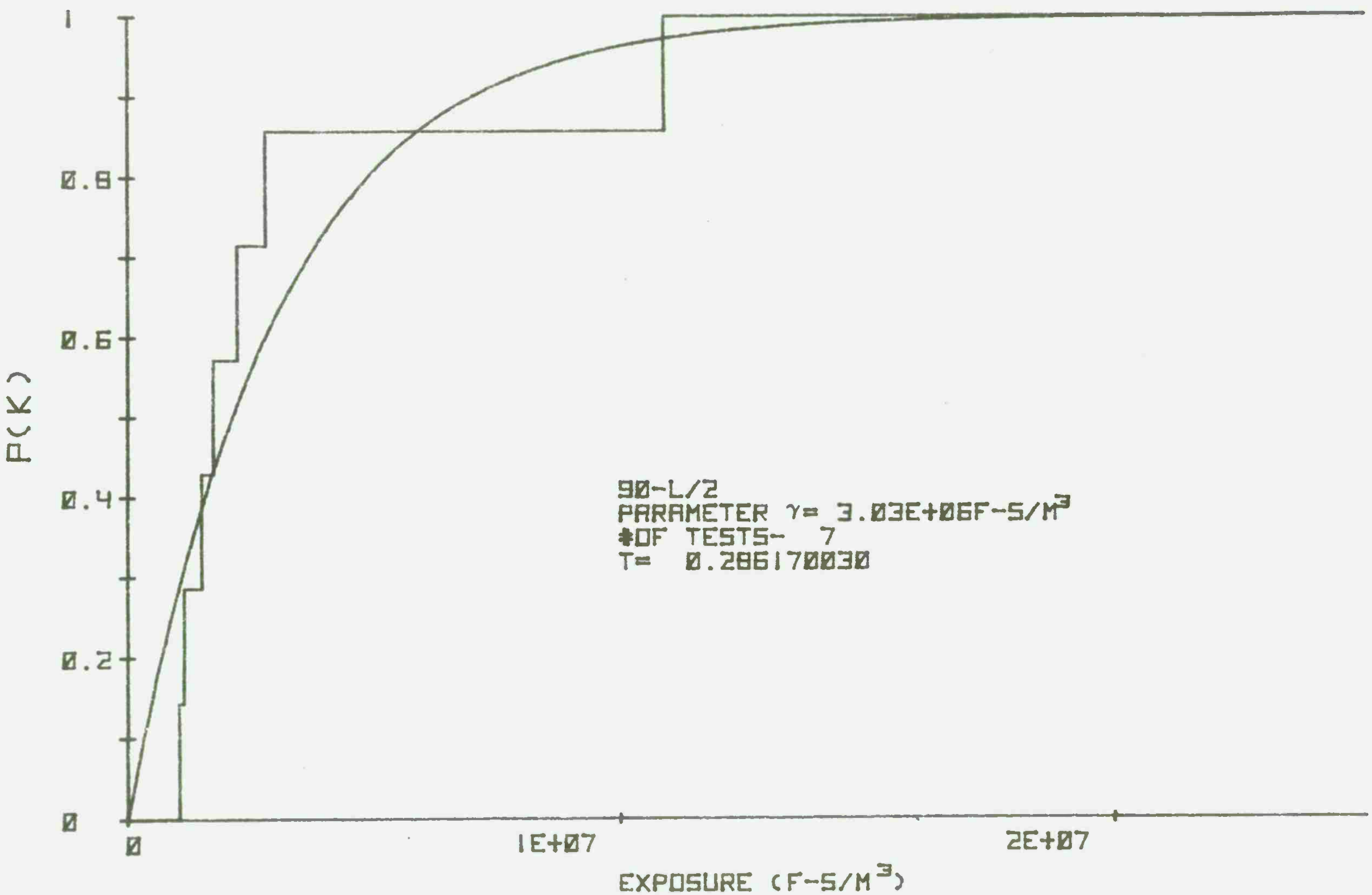

Figure 11. Plot of 90-L/2 Data and the Theoretical Distribution For a 1 Fiber Model

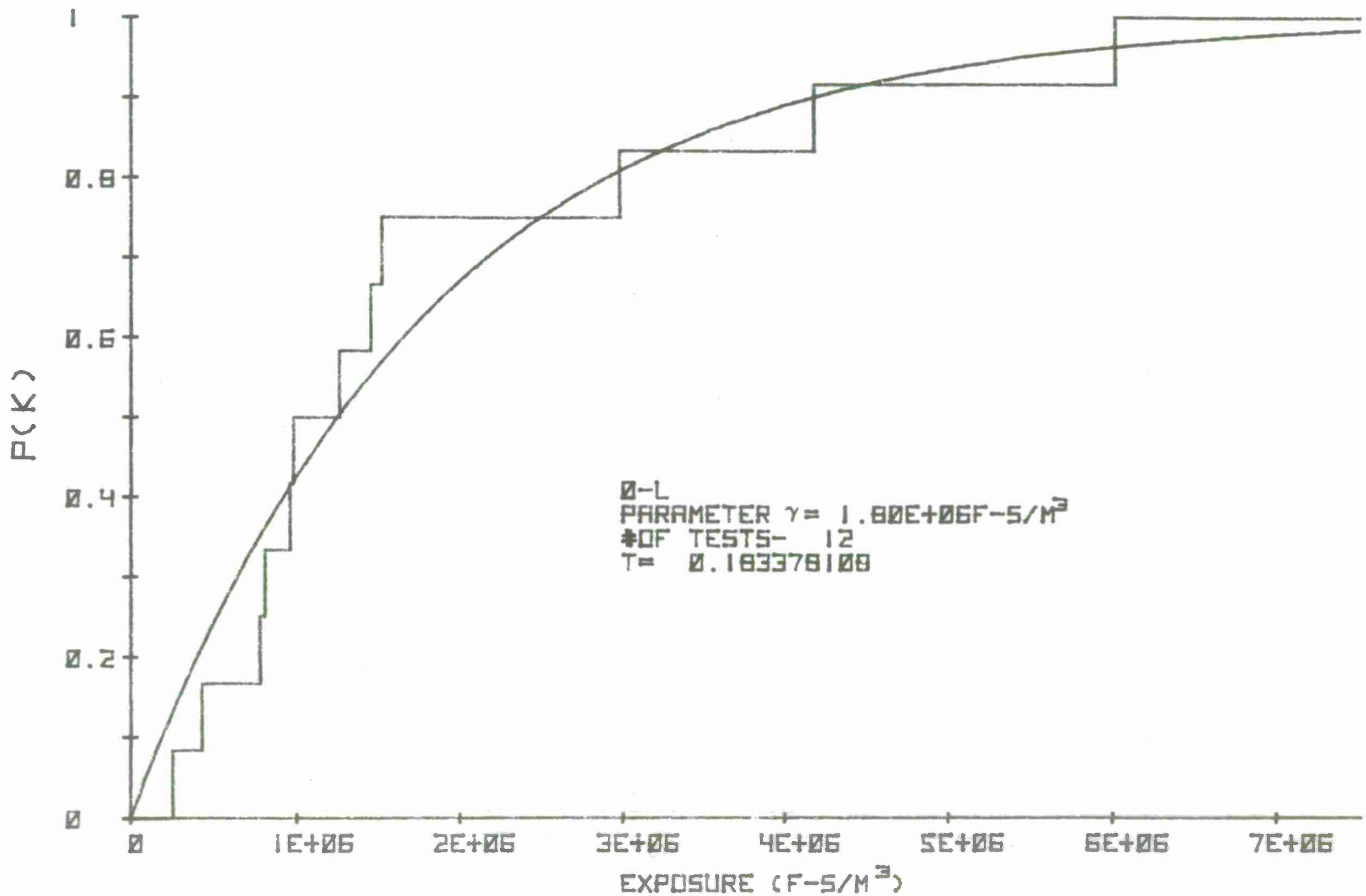

Figure 12. Plot of 0-L Data and the Theoretical Distribution For a 1 Fiber Model

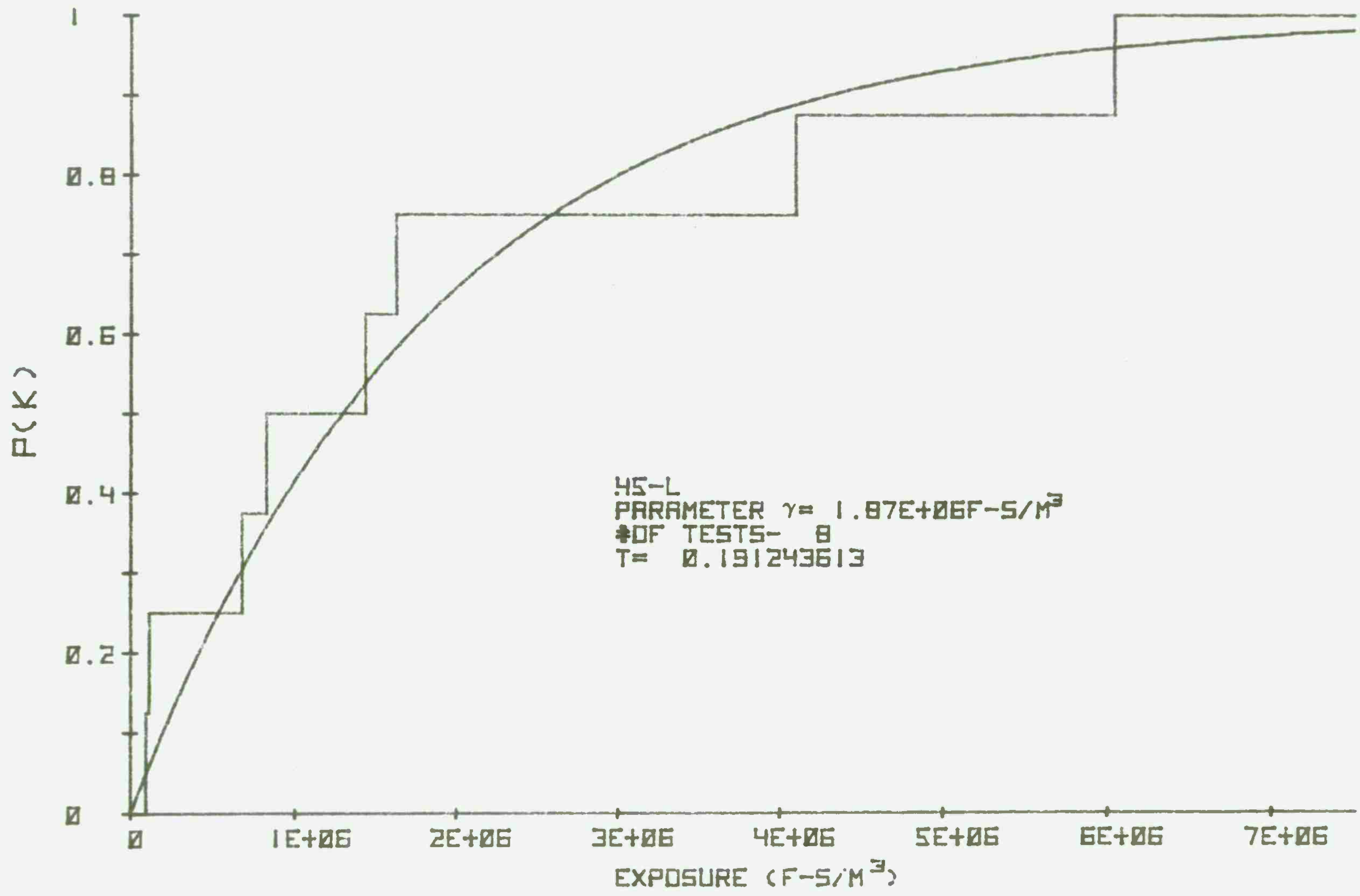

Figure 13. Plot of 45-L Data and the Theoretical Distribution For a 1 Fiber Model

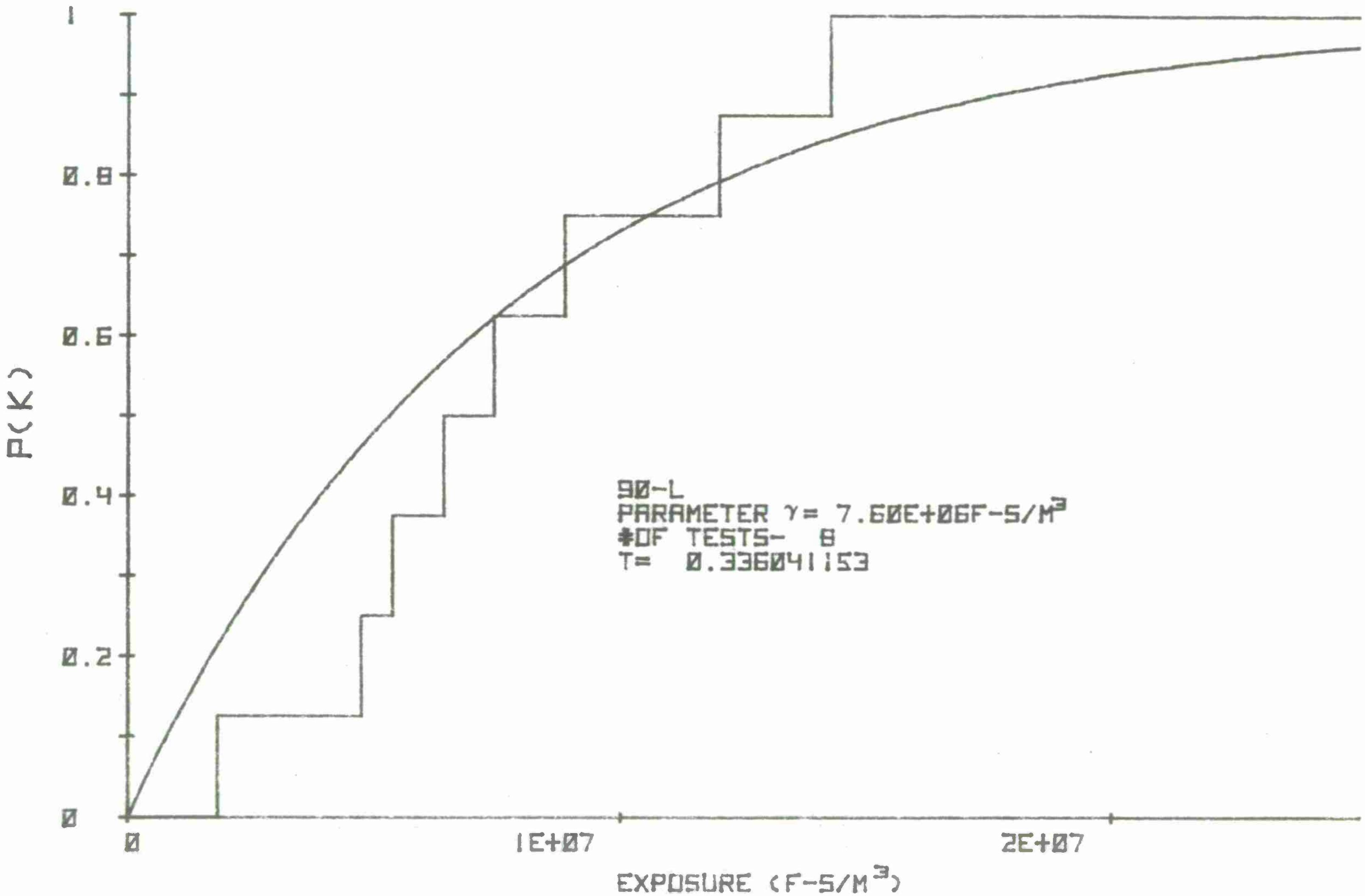

Figure 14. Plot of 90-L Data and the Theoretical Distribution For a 1 Fiber Model

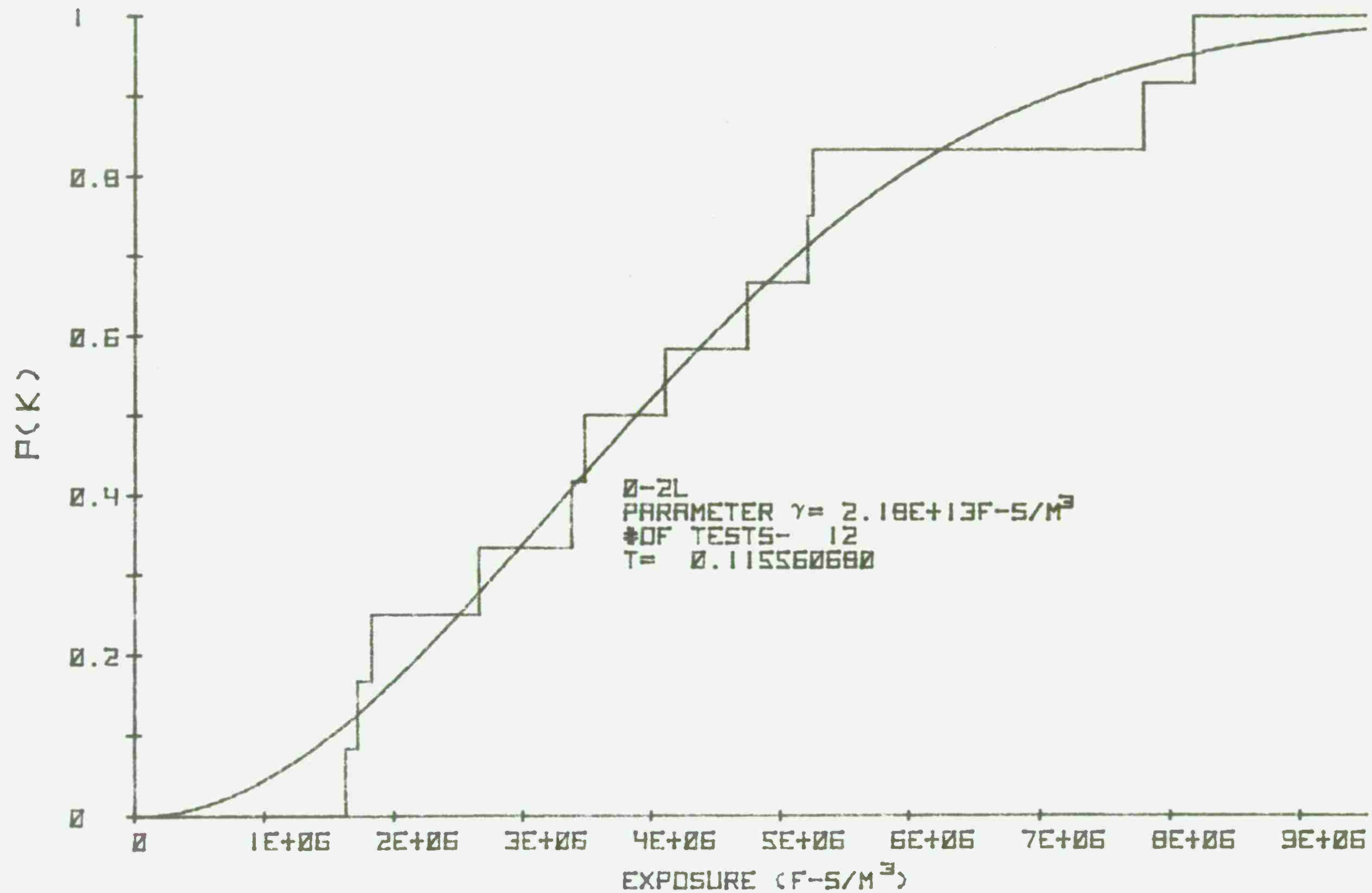

Figure 15. Plot of 0-2L Data and the Theoretical Distribution For a 2 Fiber Model

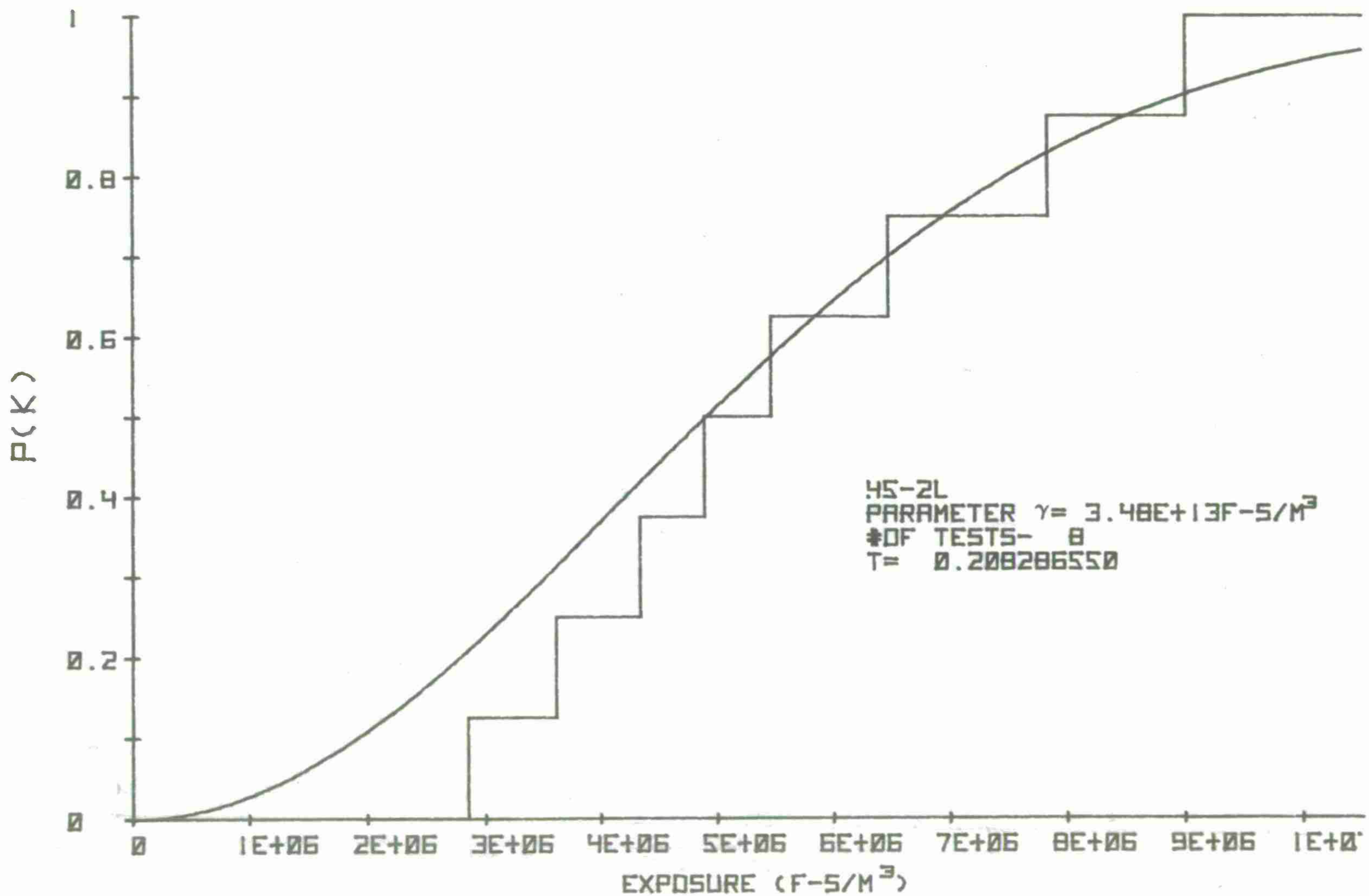

Figure 16. Plot of 45-2L Data and the Theoretical Distribution For a 2 Fiber Model

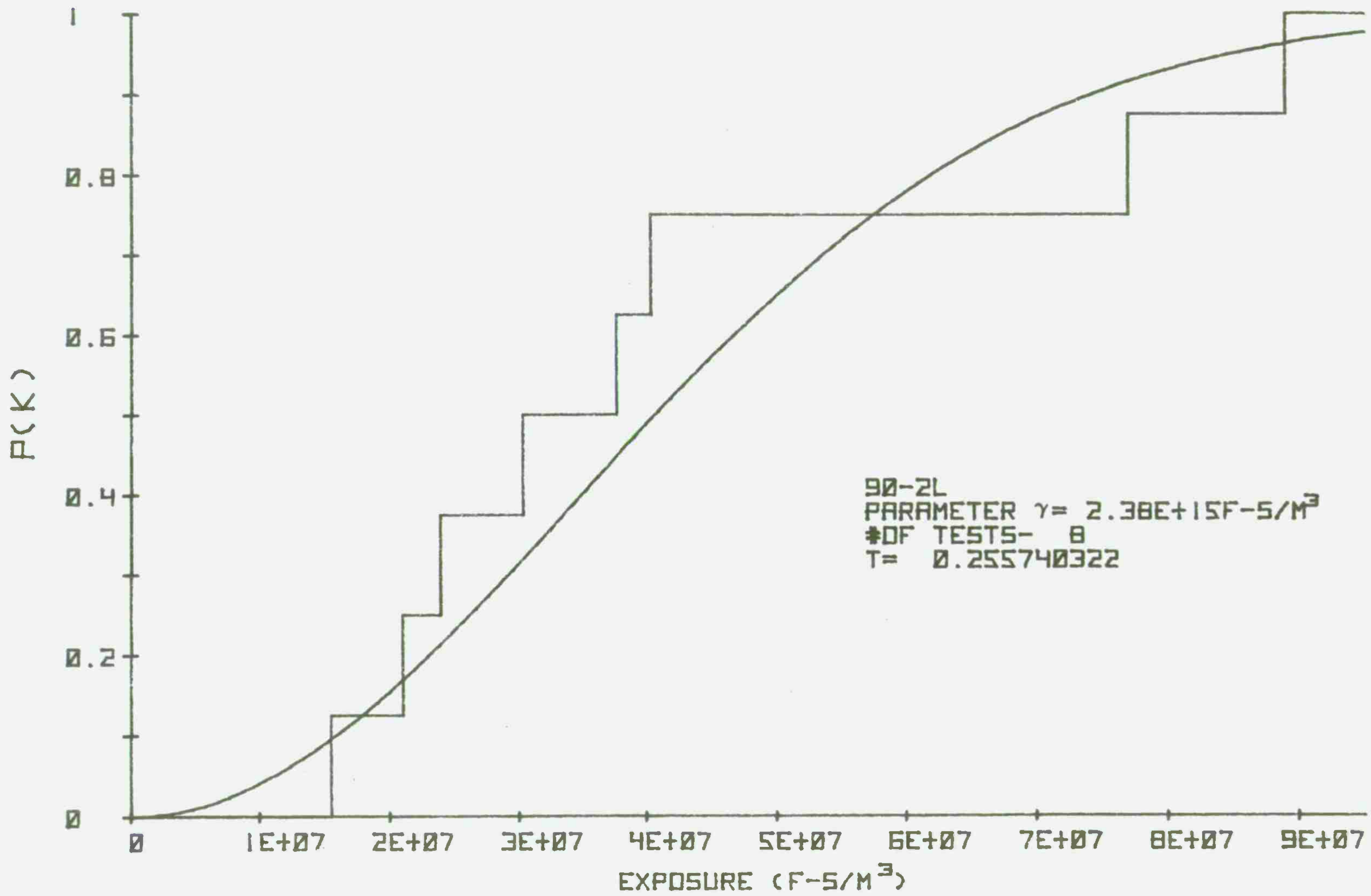

Figure 17. Plot of 90-2L Data and the Theoretical Distribution For a 2 Fiber Model

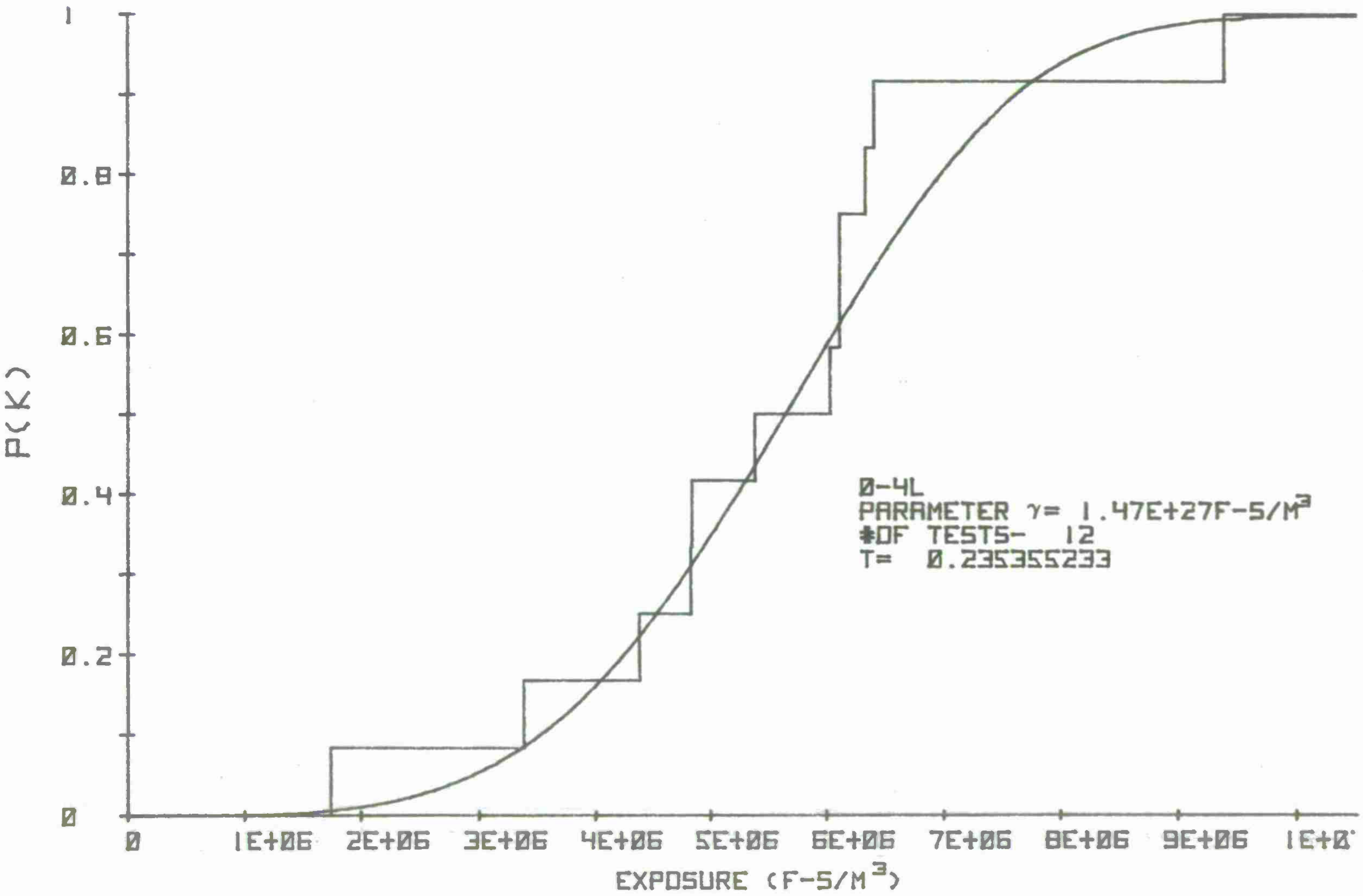

Figure 18. Plot of 0-4L Data and the Theoretical Distribution For a 4 Fiber Model

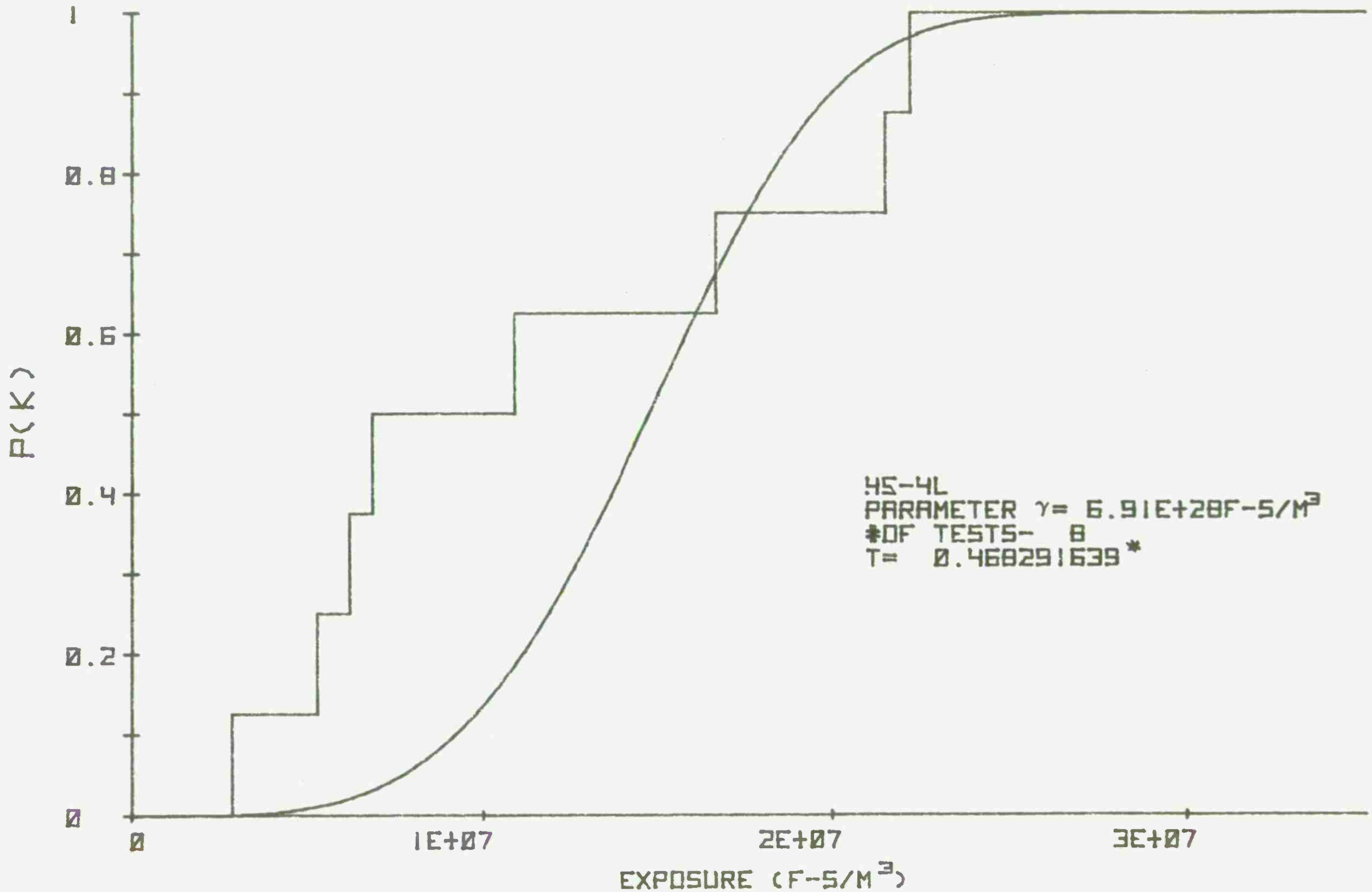

Figure 19. Plot of 45-4L Data and the Theoretical Distribution For a 4 Fiber Model

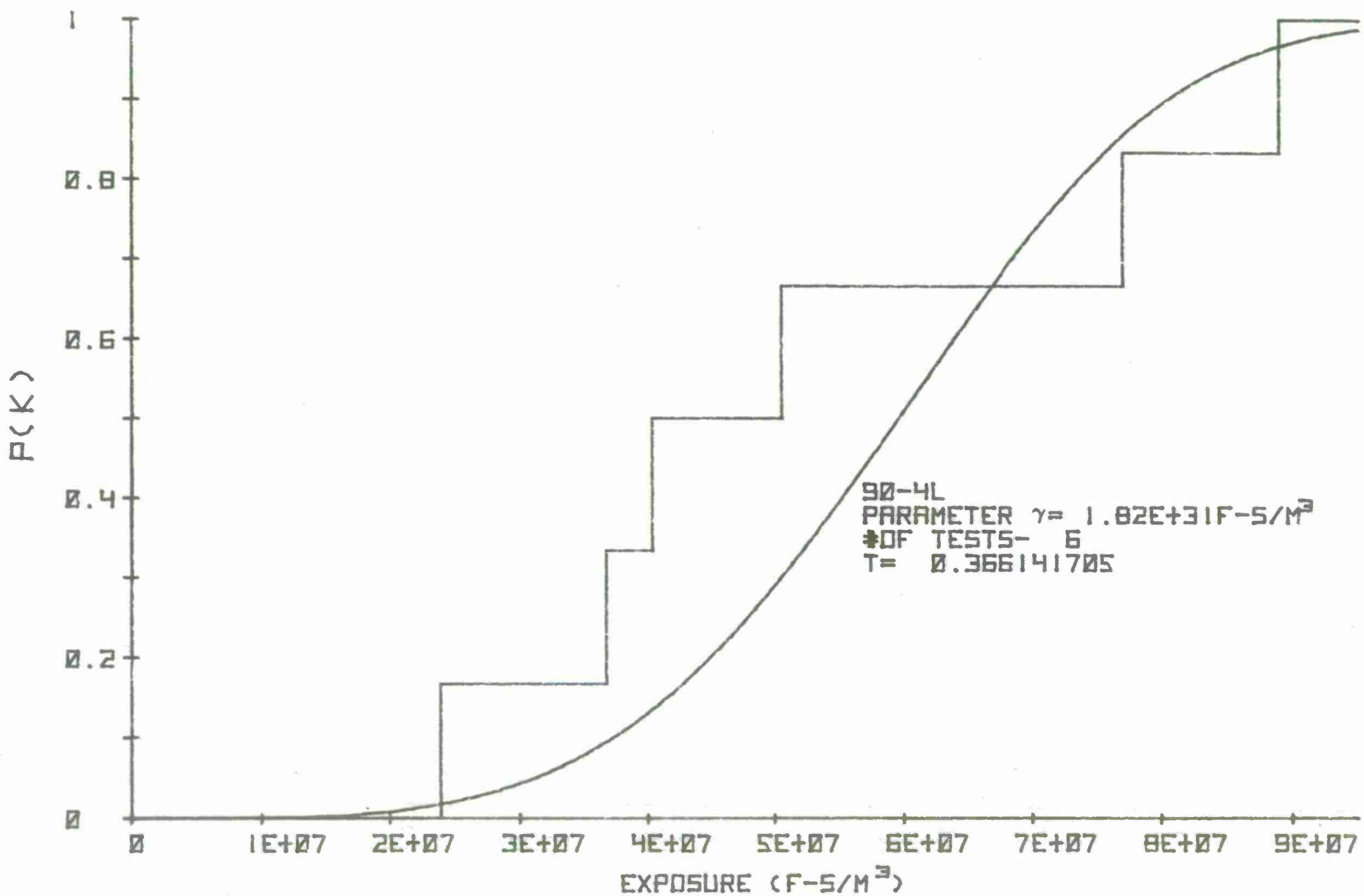

Figure 20. Plot of 90-4L Data and the Theoretical Distribution For a 4 Fiber Model

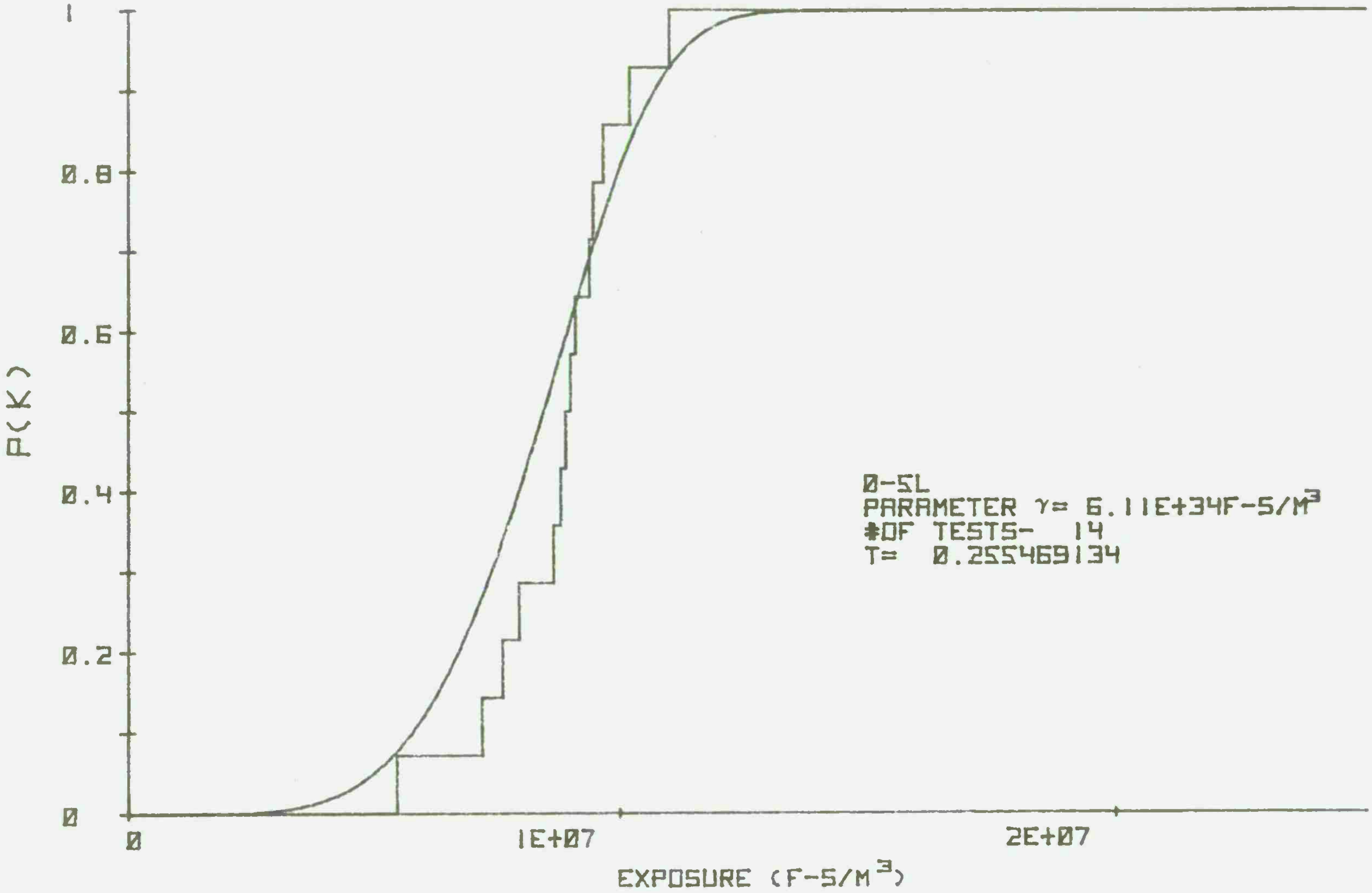

Figure 21. Plot of 0-5L Data and the Theoretical Distribution For a 5 Fiber Model

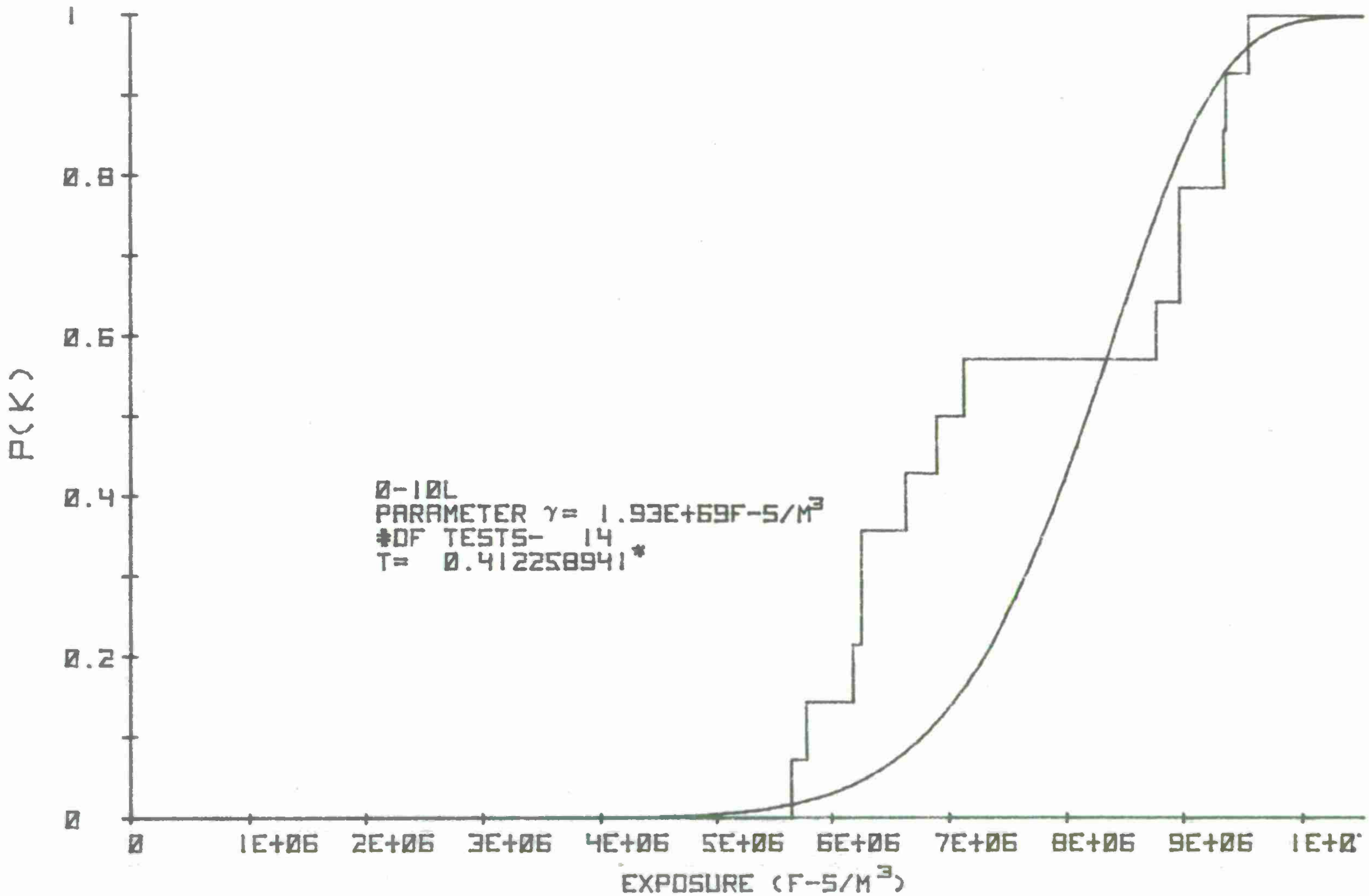

Figure 22. Plot of 0-10L Data and the Theoretical Distribution For a 10 Fiber Model

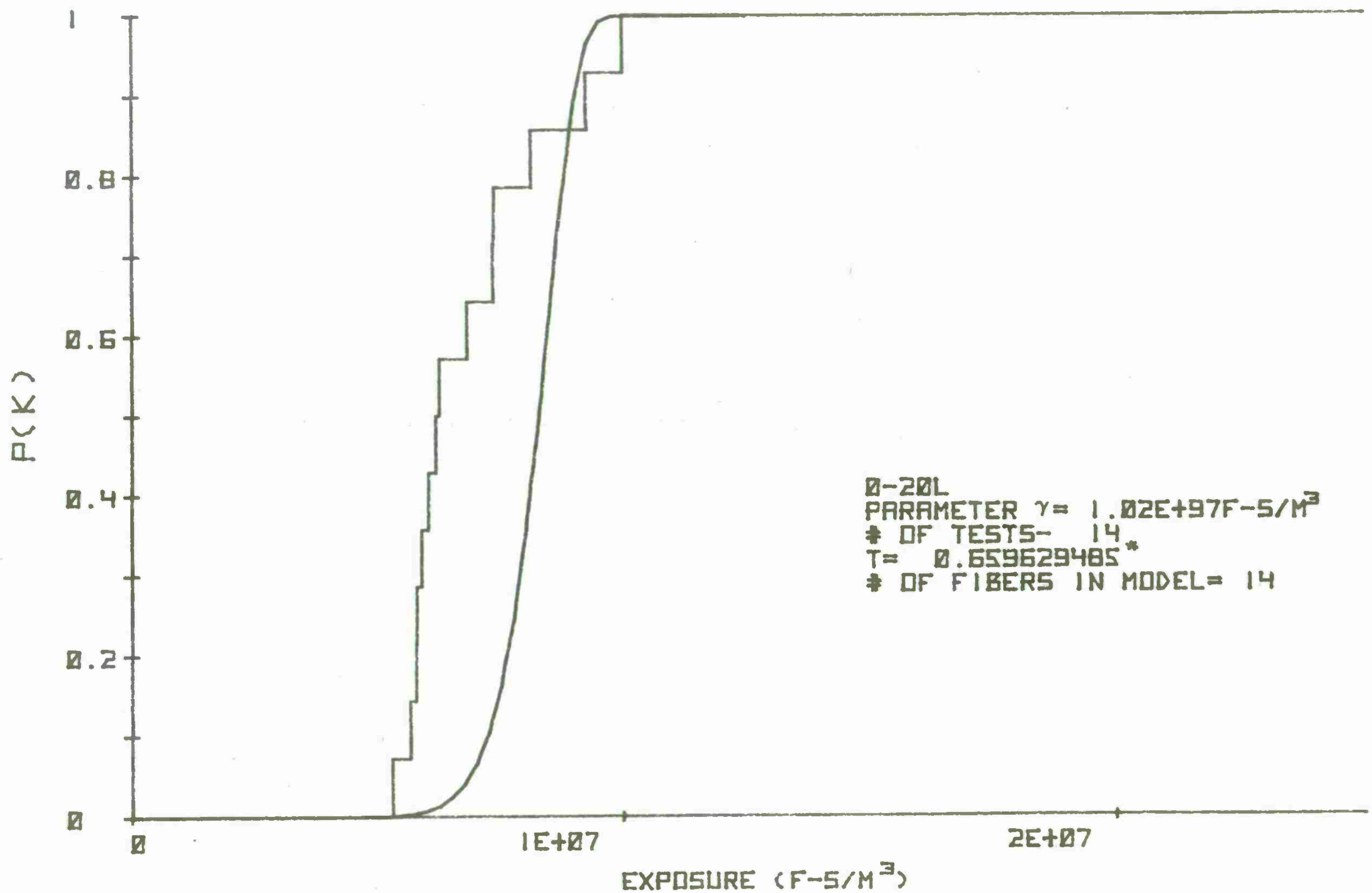

Figure 23. Plot of 0-20L Data and the Theoretical Distribution For a 14 Fiber Model

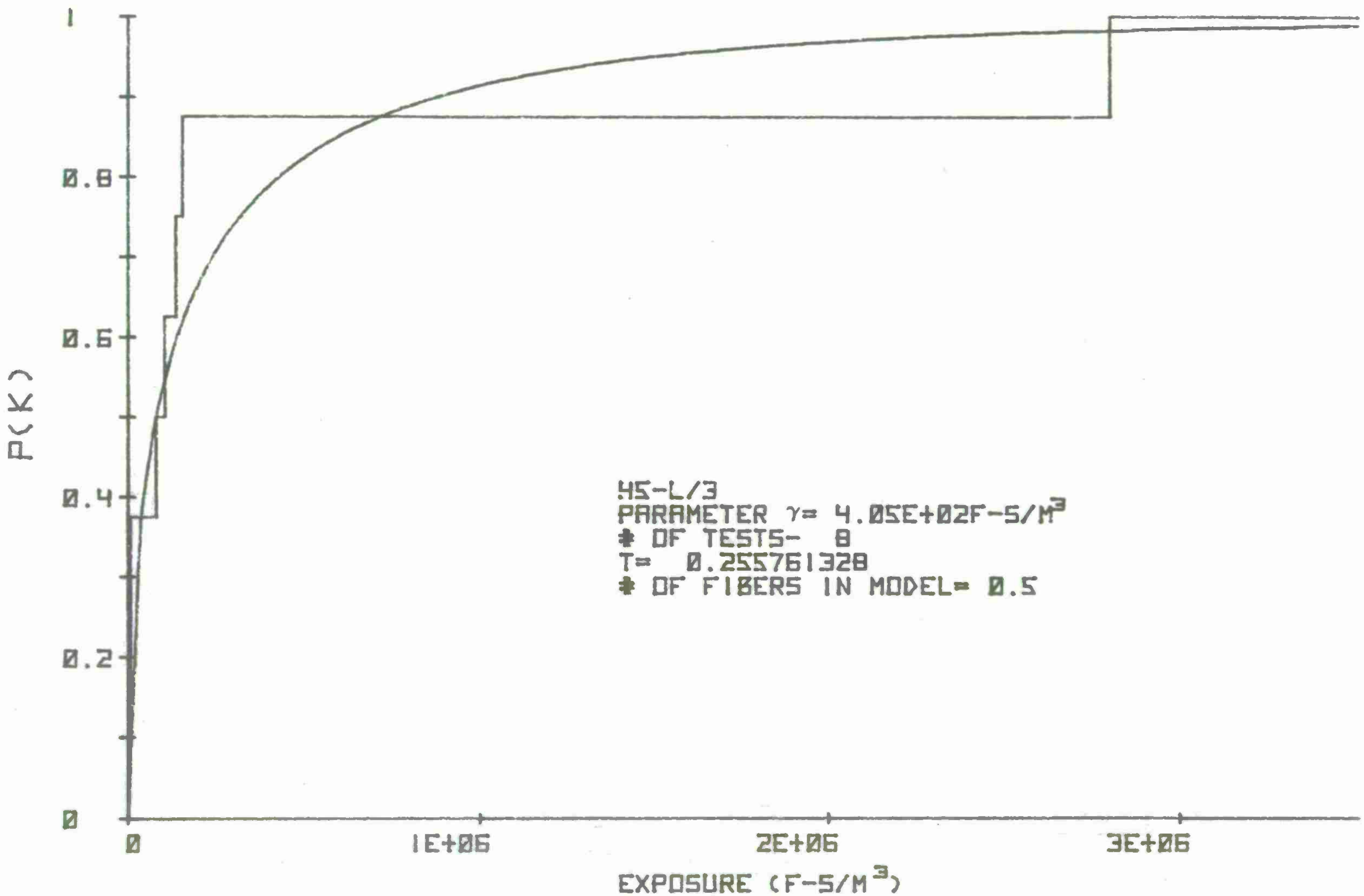

Figure 24. Plot of the 45-L/3 Data and the Theoretical Distribution For a 0.5 Fiber Model

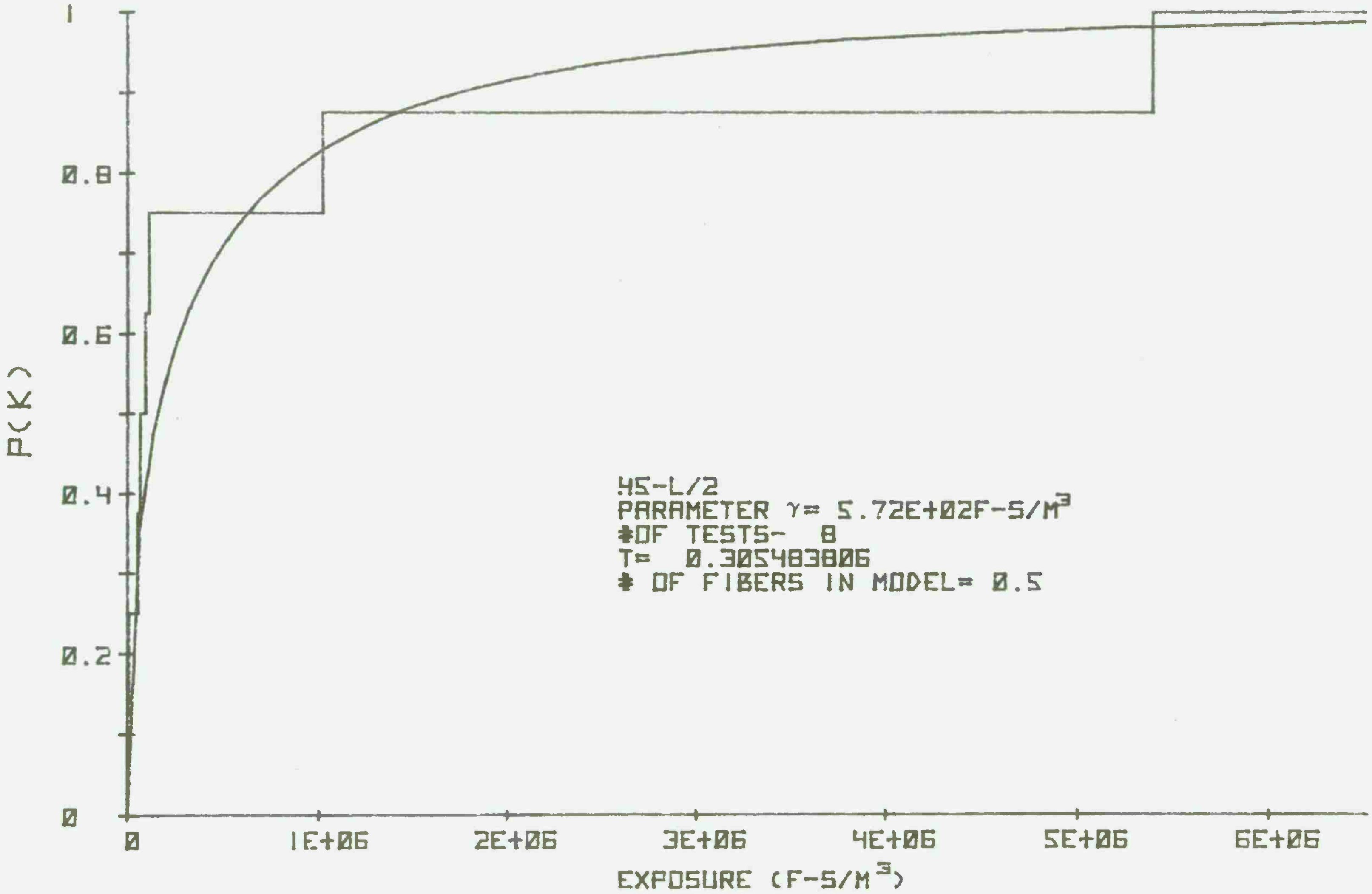

Figure 25. Plot of the 45-L/2 Data and the Theoretical Distribution For a 0.5 Fiber Model

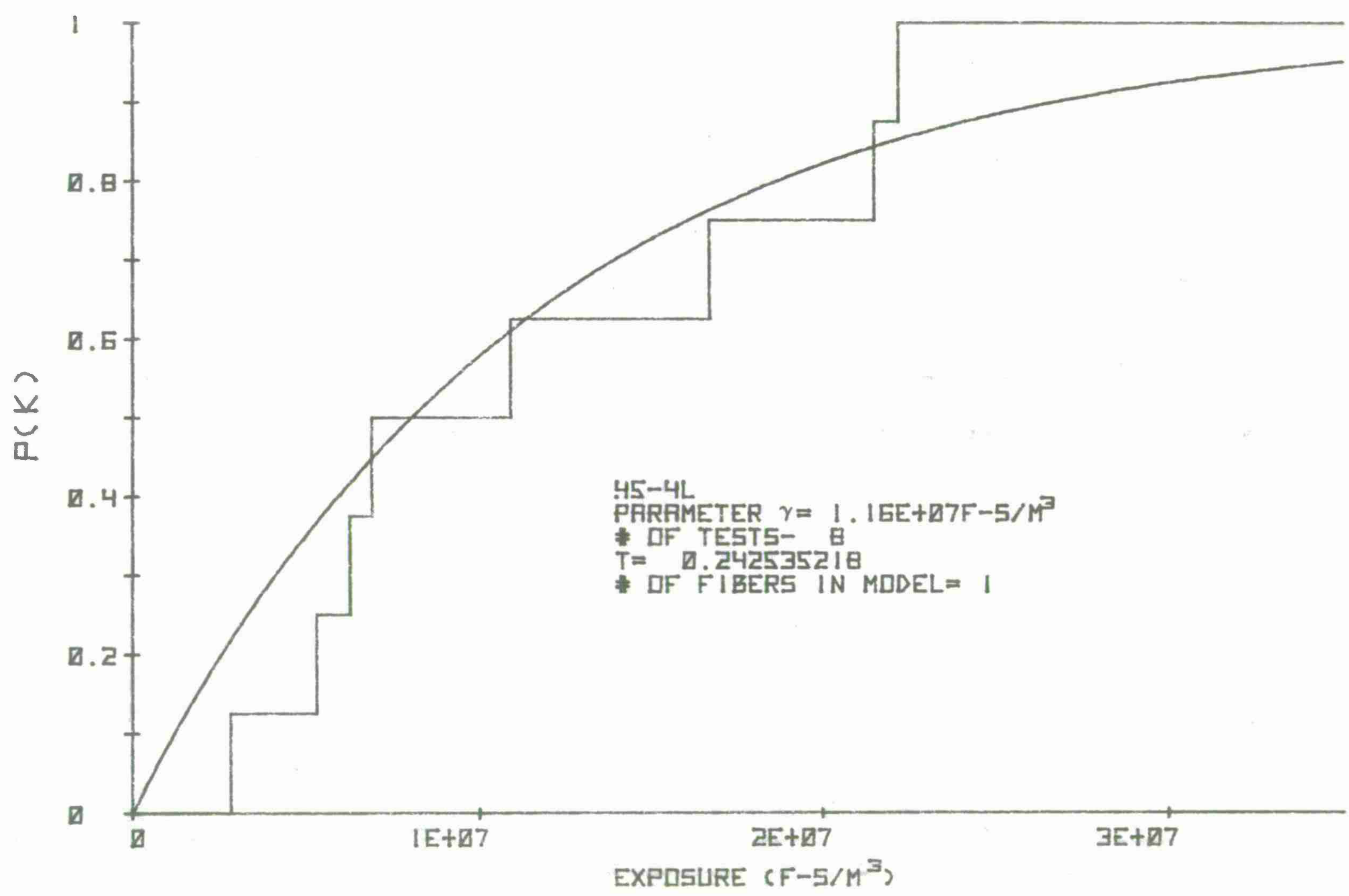

Figure 26. Plot of the 45-4L Data and the Theoretical Distribution For a 1 Fiber Model

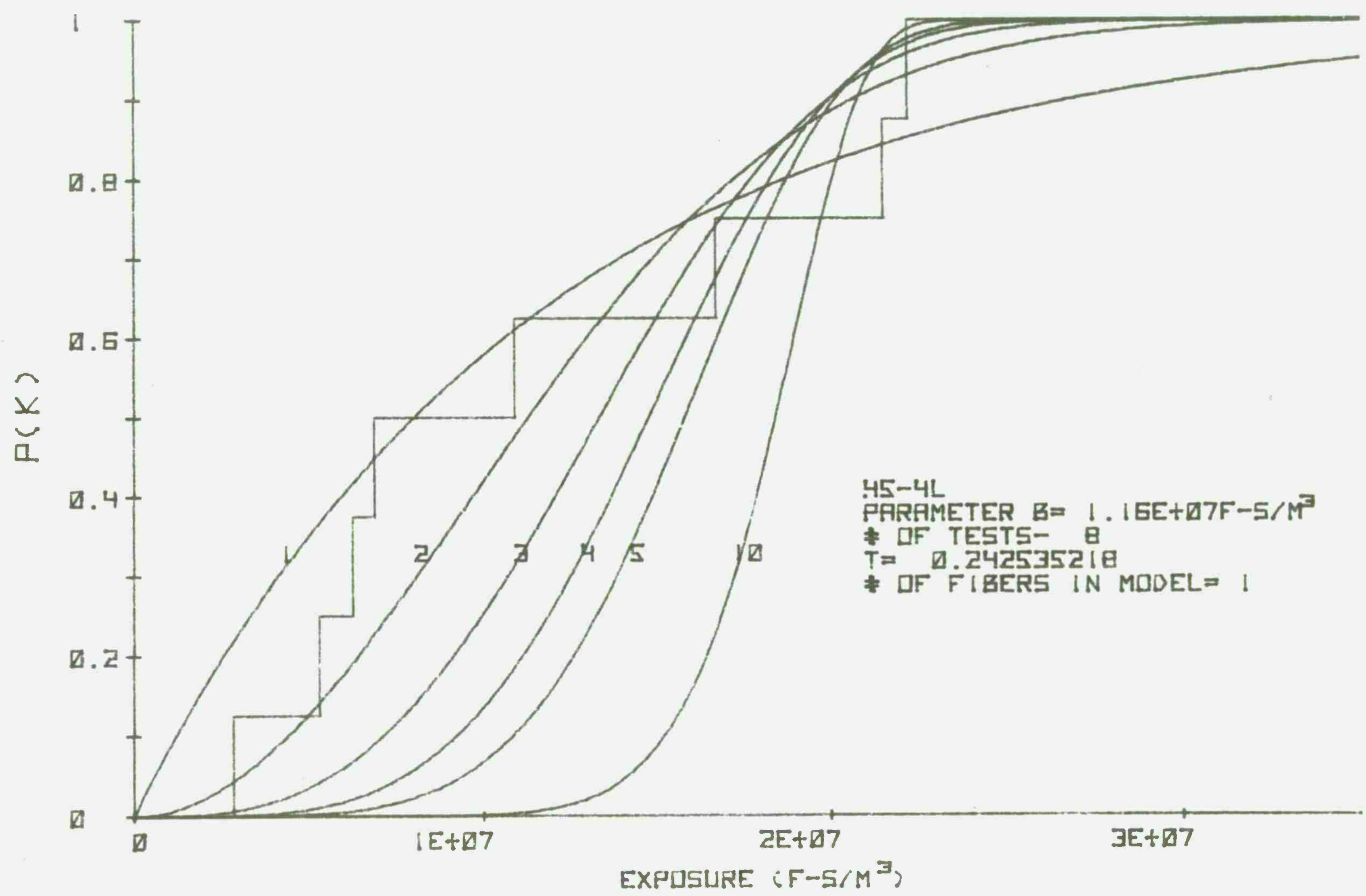

Figure 27. Plot of the 45-4L Data and the Family of Theoretical Distributions

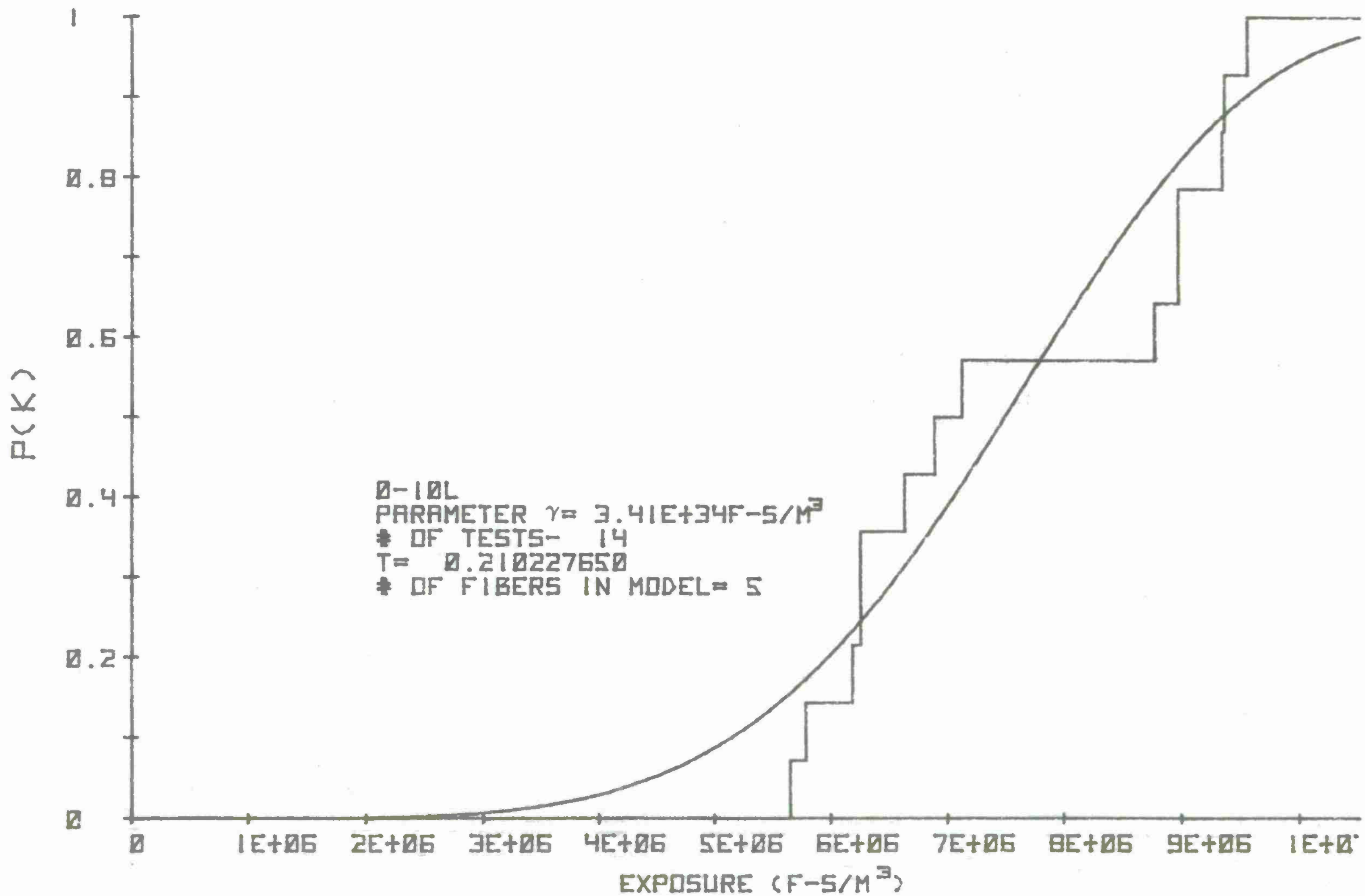

Figure 28. Plot of the 0-10L Data and the Theoretical Distribution For a 5 Fiber Model

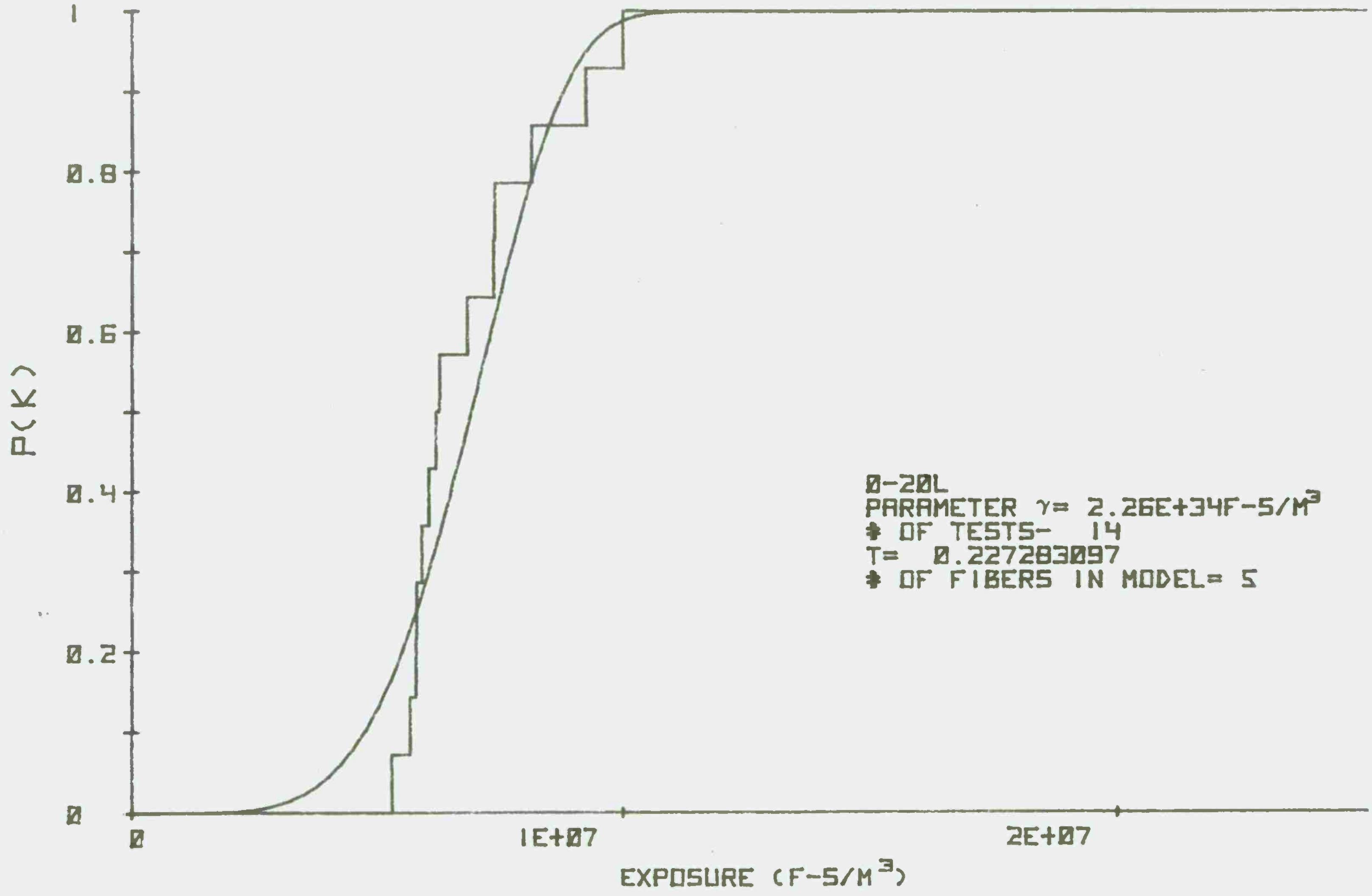

Figure 29. Plot of the 0-20L Data and the Theoretical Distribution For a 5 Fiber Model

having a large (i.e. $\geq$ 5) gap width to fiber length ratio.

VI. CONCLUSIONS

The Weibull distribution

$$F(E) = 1-e^{-\gamma E^n}$$

where n is the number of fibers needed to bridge the electrode gap, and γ is estimated from the data using the method of maximum likelihood is a very acceptable and even desirable model because of its simplicity in describing failures due to carbon fibers. When n = 1, we have an exponential distribution which is widely accepted in the case of a single fiber kill.

From the tests with gap width greater than or equal to five times the fiber length, the limiting distribution appears to be a five fiber model. That is, the situation in which a target is essentially covered by fibers in order to effect a kill is best fit by a five fiber model.

ACKNOWLEDGEMENT

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APPENDIX A

Maximum Likelihood Estimate of the Scale Parameter for a Two Parameter Weibull Distribution

The likelihood function, L, is the likelihood that a random variable E assumes particular values E_1, E_2, ... , E_r, that is, the probability of obtaining a sample with outcomes (exposures to failure) E_1, E_2, ..., E_r. In particular, if E_1, E_2, ..., E_r is a random sample from the density of $f(E;\gamma)$, then the likelihood function is

$$L = f(E_1;\, \gamma)\ f\,(E_2;\, \gamma)\ \ldots\ f\,(E_r;\, \gamma).$$

In our tests we have two possible outcomes; 1) a failure is observed at exposure E_i or 2) no failure occurs and the test is terminated at exposure E_i.

The probability to failure distribution in which we are interested is

$$F_n(E_i) = 1-\exp(-\gamma E_i^{\,n})$$

The density function associated with a failure is therefore

$$f_n\ (E_i) = n\gamma E_i^{\,n-1}\ \exp\ (-\gamma E_i^{\,n}).$$

The density function associated with a test terminated (survival) at E_i is

$$f_n\ (E_i) = \exp\ (-\gamma E_i^{\,n}).$$

Therefore, if in a sample of r tests there are k failures, the likelihood function is

$$L = \left[n^k\ \gamma^k \left(\prod_{i=1}^{k}\ (E_i^{\,n-1}) \right) \exp\left(-\gamma \sum_{i=1}^{k}\ E_i^{\,n}\right) \right] \left[\exp\ \left(-\gamma \sum_{i=k+1}^{r}\ E_i^{\,n}\right) \right]$$

$$= n^k\ \gamma^k \left(\prod_{i=1}^{k}\ (E_i^{\,n-1}) \right) \exp\left(-\gamma \sum_{i=1}^{r}\ E_i^{\,n}\right).$$

Maximizing the natural log of the likelihood function is equivalent to maximizing the likelihood function itself, so that

$$\mathcal{L} = \ln L = k\ (\ln n) + k\ (\ln \gamma) + \sum_{i=1}^{k} (\ln E_i^{\,n-1} - \gamma \sum_{i=1}^{r} (E_i^{\,n})$$

$$\frac{d\mathcal{L}}{d\gamma} = \frac{k}{\gamma} - \sum_{i=1}^{r} (E_i^{\,n})$$

$$0 = \frac{k}{\hat{\gamma}} - \sum_{i=1}^{r} (E_i^{\,n})$$

$$\hat{\gamma} = k \Big/ \sum_{i=1}^{r} (E_i^{\,n})$$

$\hat{\gamma}$ is the maximum likelihood estimate of γ and $1/\hat{\gamma}$ is called the mean exposure to failure.

APPENDIX B

Distribution of the Kolmolgorov-Smirnov Statistic for the Weibull Distribution with Scale Parameter Unknown

The Weibull distribution derived in Section II is

$$F(E) = 1-\exp[-\gamma E^n].$$

For a given n, and γ estimated from the data using the method of maximum likelihood, the distribution of the Kolmolgorov-Smirnov (K-S) Statistic will be determined.

A set of uniform random numbers, R_i , are generated and solving the following equation a set of exposures to failure, E_i , are determined for a given n and γ

$$R_i = \exp[-\gamma E_i{}^n]$$

and $$E_i = \left(\frac{1}{-\gamma} \ln R_i\right)^{1/n} \qquad \text{B.1}$$

Since γ is unknown, it is necessary to estimate γ from the data. Using the method of maximum likelihood (Appendix A)

$$\hat{\gamma} = k / \sum_{i=1}^{r} E_i{}^n$$

where r is the number of tests and k is the number of failures. Substituting E_i from Eq. B.1, we have

$$\hat{\gamma} = -\gamma k / \ln \prod_{i=1}^{r} R_i$$

Therefore,

$$\hat{F}(E) = 1-\exp\,[-\hat{\gamma} E^n]$$

$$= 1-\exp\,[\gamma k E^n / \ln \prod_{i=1}^{r} R_i]$$

The K-S statistic, T, is defined as follows:

$$T_i = |S\,(E_{(i)}) - F\,(E_{(i)})|$$

$$T_i^- = \left| S(E_{(i-1)}) - F(E_{(i)}) \right|$$

$$\text{and } T = \max_{i=1,\ldots,k} \left\{ T_i,\ T_i^- \right\}$$

where $S(E_i)$ is the empirical distribution function and $S(E_0) = 0$ by definition. The subscript (i) indicates the ith order statistic.

Note: $R_i<1$, $E_i > 0$, and $\ln R_i < 0$. $E_1 \leq E_2 \leq \ldots \leq E_k$ and $E_i < E_j$ implies $R_i > R_j$ so that the rankings on the E_i's and R_i's are in reverse order.

$$T_i = \left| S(E_{(i)}) - F(E_{(i)}) \right|$$

$$= \left| \frac{i}{k} - \left[1 - \exp\left[\gamma k (E^n_{(i)}) / \ln \left(\prod_{i=1}^{r} R_i \right) \right] \right] \right|$$

$$= \left| \frac{i-k}{k} + \exp\left[\gamma k (E^n_{(i)}) / \ln \left(\prod_{i=1}^{r} R_i \right) \right] \right|$$

Similarly,

$$T_i^- = \left| \frac{i-1-k}{k} + \exp\left[\gamma k (E^n_{(i)}) / \ln \left(\prod_{i=1}^{r} R_i \right) \right] \right|$$

$$P\left\{T > T^*\right\} = P\left\{ \max_i \left\{ T_i,\ T_i^- \right\} > T^* \right\}$$

$$= 1 - P\left\{ T_i \leq T^* \cap T_i^- \leq T^*,\ \forall\ i, i- \right\}$$

$$P\left\{ T_i \leq T^* \right\} = P\left\{ \left| \frac{i-k}{k} + \exp\left[\gamma k E^n_{(i)} \Big/ \ln \left(\prod_{i=1}^{r} R_i \right) \right] \right| \leq T^* \right\}$$

Solving for E_i, we have $P\left\{ T_i \leq T^* \right\} =$

$$P\left\{ \left[\frac{\left(\ln \prod_{i=1}^{r} R_i \right) \left[\ln \left(\frac{k-i}{k} - T^* \right) \right]}{\gamma k} \right]^{1/n} \geq E_{(i)} \geq \left[\frac{\left(\ln \prod_{i=1}^{r} R_i \right) \left[\ln \left(\frac{k-i}{k} + T^* \right) \right]}{\gamma k} \right]^{1/n} \right\}$$

Substituting for $E_{(i)}$ from Eq. B.1 and using the fact that the E_i's and R_i's are in reverse order.

$$P\left\{T_i \leq T^*\right\} = P\left\{\left[\frac{\left(\ln \prod_{i=1}^{r} R_i\right)\left[\ln\left(\frac{k-i}{k} - T^*\right)\right]}{\gamma k}\right]^{1/n} \geq \left(\frac{-1}{\gamma} \ln R_{(k+1-i)}\right)^{1/n}\right.$$

$$\left. \geq \left[\frac{\left(\ln \prod_{i=1}^{r} R_i\right)\left[\ln\left(\frac{k-i}{k} + T^*\right)\right]}{\gamma k}\right]^{1/n}\right\}$$

Therefore,

$$P\left\{T_i \leq T^*\right\} = P\left\{\frac{\left(\ln \prod_{i=1}^{r} R_i\right)\left[\ln\left(\frac{k-i}{k} - T^*\right)\right]}{k} \geq -\ln R_{(k+1-i)} \geq \right.$$

$$\left. \frac{\left(\ln \prod_{i=1}^{r} R_i\right)\left[\ln\left(\frac{k-i}{k} + T^*\right)\right]}{k}\right\}$$

Using a similar argument

$$P\left\{T_{i-} \leq T^*\right\} = P\left\{\frac{\left(\ln \prod_{i=1}^{r} R_i\right)\left[\ln\left(\frac{k+1-i}{k} - T^*\right)\right]}{k} \geq -\ln R_{(k+1-i)} \geq \right.$$

$$\left. \frac{\left(\ln \prod_{i=1}^{r} R_i\right)\left[\ln\left(\frac{k+1-i}{k} + T^*\right)\right]}{k}\right\}$$

Therefore, the distribution of the T statistic of the Weibull distribution with the scale parameter estimated from the data is independent of the shape parameter, n, and the tables of the distribution of the T statistic for the exponential distribution with mean unknown [6] are appropriate.

---

6 *Hubert W. Lilliefors, "On the Kolmogorov-Smirnov Test for the Exponential Distribution with Mean Unknown", JASA, March,1969, pp. 387-389.*

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